Planning is hot...

- Significant scale-up in the last 4-5 years
  - Before we could synthesize about 5-6 action plans in minutes
  - Now, we can synthesize 100-action plans in minutes
  » Further scale-up with domain-specific control
- Significant strides in our understanding
  - Rich connections between planning and CSP(SAT) OR (ILP)
    » Vanishing separation between planning & Scheduling
  - New ideas for heuristic control of planners
  - Wide array of approaches for customizing planners with domain-specific knowledge
Overview

- The Planning problem
  - Our focus
  - Modeling, Proving correctness
- Refinement Planning: Formal Framework
- Conjunctive refinement planners
- Disjunctive refinement planners
  - Refinement of disjunctive plans
  - Solution extraction from disjunctive plans
    » Direct, Compiled (SAT, CSP, ILP, BDD)
- Heuristics/Optimizations
- Customizing Planners
  - User-assisted Customization
  - Automated customization
- Support for non-classical worlds

Planning: The big picture

- Synthesizing goal-directed behavior
- Planning involves
  - Action selection; Handling causal dependencies
  - Action sequencing and handling resource allocation
    » typically called SCHEDULING
  - Depending on the problem, plans can be
    » action sequences
    » or “policies” (action trees, state-action mappings etc.)
The Many Complexities of Planning

Environment

Goals

What action next?

Static vs. Dynamic

(Stochastic vs. Deterministic)

(Observable vs. Partially Observable)

(Full vs. Partial satisfaction)

(Deterministic vs. Stochastic)

(perfect vs. imperfect)

(action)

perception

The Many Complexities of Planning

Dynamic

Stochastic

Partially Observable

Durable

Continuous

Static

Deterministic

Observable

Instantaneous

Propositional

"Classical Planning"
Broad Aims & Biases of the Tutorial

**AIM:** We will concentrate on planning in deterministic, quasi-static, and fully observable worlds. Will start with "classical" domains; but discuss handling durative actions and numeric constraints, as well as replanning.

**BIAS:** To the extent possible, we shall shun brand-names and concentrate on unifying themes:
- Better understanding of existing planners
- Normalized comparisons between planners
- Evaluation of trade-offs provided by various design choices
- Better understanding of inter-connections
- Hybrid planners using multiple refinements
- Explication of the connections between planning, CSP, SAT and ILP

Why Care about “neo-classical” Planning?

- Most of the recent advances occurred in neo-classical planning.
- Many stabilized environments satisfy neo-classical assumptions:
  - It is possible to handle minor assumption violations through replanning and execution monitoring.
  - This form of solution has the advantage of relying on widely-used (and often very efficient) classical planning technology. Bouillon, 2000.
- Techniques developed for neo-classical planning often shed light on effective ways of handling non-classical planning worlds:
  - Currently, most of the efficient techniques for handling non-classical scenarios are still based on ideas/advances in classical planning.
Applications (Current & Potential)

- Scheduling problems with action choices as well as resource handling requirements
  - Problems in supply chain management
  - HSTS (Hubble Space Telescope scheduler)
  - Workflow management
- Autonomous agents
  - RAX/PS (The NASA Deep Space planning agent)
- Software module integrators
  - VICAR (JPL image enhancing system); CELWARE (CELCorp)
  - Test case generation (Pittsburgh)
- Interactive decision support
  - Monitoring subgoal interactions
    » Optimum AIV system
- Plan-based interfaces

The (too) many brands of classical planners

Planning as Theorem Proving (Green’s planner) → Planning as Search

Planning as Model Checking

Search in the space of States (progression, regression, MEA)
  (STAIRS, PRODIGY, TOP, HSP, HSP-R, UNPOP, FF)

Search in the space of Plans (total order, partial order, protections, MTC)
  (Interplan, SNLP, TOCL, UCPOP, TWEAK)

Planning as CSP/ILP/SAT/BDD (Graphplan, IPP, STAN, SATPLAN, BLackBOX, GP-CSP, BDDPlan)
A Unifying View

**PART 1**

1. Refinement Planning
   - What are Plans? [Candidate set semantics]
   - How are plans represented compactly?
   - How are they refined?
   - How are they searched?

   **Conjunctive Refinement Planning**
   - Graph-based
   - CSP
   - SAT
   - ILP
   - BDD

   **Disjunctive Refinement Planning**

**PART 2**

- Directed Partial Consistency enforcement
- Reachability
- Relevance
- Relax Subgoal interactions
- Heuristics/Optimizations

**PART 3**

- HTN Schemas
- TL Formulas
- Cutting Planes
- Case-based
- Abstraction-based
- Failure-based
- Domain Analysis

Overlap with other courses

- Hector will talk more on heuristics
- Dana & Derek will talk more on domain-specific knowledge
- Paolo will talk more on model-checking
- Malik will talk more on temporal planning
- Bernhard will talk more on the formal stuff

I want to know God’s thoughts... The rest is mere details.

Albert Einstein
Modeling Classical Planning

- States are modeled in terms of (binary) state-variables
  - Complete initial state, partial goal state
- Actions are modeled as state transformation functions
  - Syntax: ADL language (Pednault)
  - Apply(A,S) = (S \ eff(A)) + eff(A)
    (If Precond(A) hold in S)

\[ \text{At}(A,M), \neg \text{At}(B,M) \]
\[ \neg \text{In}(A), \neg \text{In}(B) \]

\[ \forall x \text{In}(x) \Rightarrow \text{At}(x,M) \]
\[ \neg \text{At}(x,E) \]

\[ \text{Prec.} \]
\[ \text{Load}(o_1), \text{At}(o_1,l_1) \]
\[ \text{Unload}(o_1), \text{In}(o_1) \]

\[ \text{Fly}() \]
\[ \text{At}(R,E) \]
Some notes on action representation

- STRIPS Assumption: Actions must specify all the state variables whose values they change...
- No disjunction allowed in effects
  - Conditional effects are NOT disjunctive
    » (antecedent refers to the previous state & consequent refers to the next state)
- Quantification is over finite universes
  - essentially syntactic sugaring
- All actions can be compiled down to a canonical representation where preconditions and effects are propositional
  - Exponential blow-up may occur (e.g. removing conditional effects)
    » We will assume the canonical representation

Action A
Eff: If P then R
If Q then W

Action A1
Prec: P, Q
Eff: R, W

Action A2
Prec: P, ~Q
Eff: R, ~W

Action A3
Prec: ~P, Q
Eff: ~R, W

Action A4
Prec: ~P, ~Q
Eff:

Actions with Resources and Duration

Load(P:package, R:rocket, L:location)

Resources: ?h : robot hand
Preconditions: Position(?h, L) [?s, ?e]
Free(?h) ?s
Charge(?h) > 5 ?s

Effects: holding(?h, P) [?s, ?t1]
depositing(?h, P, R) [?t2, ?e]
Busy(?h) [?s, ?e]
Free(?h) ?e
Charge - = .03*(?e - ?s) ?e

Constraints: ?t1 < ?t2
?e - ?s in [1.0, 2.0]
Capacity(robot) = 3
Planning vs. Scheduling

Planning
- Initial state & a set of Goals,
- A library of actions
  » Preconditions/effects
    - Discrete/Continuous
  » Resource requirements
Synthesize a sequence of actions capable of satisfying goals

Scheduling
- Set of jobs (may have of tasks in some (partial) order)
- Temporal constraints on jobs
  » EST, LFT, Duration
- Contention constraints
  » Each task can be done on a subset of machines
Find start times for jobs that are optimal (wrt make-spans, resource consumption etc)

Checking correctness of a plan: The State-based approaches

- Progression Proof: Progress the initial state over the action sequence, and see if the goals are present in the result

- Regression Proof: Regress the goal state over the action sequence, and see if the initial state subsumes the result
Checking correctness of a plan: The Causal Approach

Causal Proof: Check if each of the goals and preconditions of the action are

» “established”: There is a preceding step that gives it

» “unclobbered”: No possibly intervening step deletes it

- Or for every preceding step that deletes it, there exists another step that precedes the conditions and follows the deleter adds it back.

Causal proof is

- “local” (checks correctness one condition at a time)
- “state-less” (does not need to know the states preceding actions)
  - Easy to extend to durative actions
- “incremental” with respect to action insertion
  - Great for replanning

The Refinement Planning Framework:

1. Syntax & Semantics of partial plans
2. Refinement strategies & their properties
3. The generic Refinement planning template
**Refinement Planning: Overview**

- **All Solutions**
- **All Action Sequences**
- **Partial Plans**
- **Refinements**

Narrowing *sets of action sequences* to progress towards solutions

Remove non-solutions

---

**Partial Plans: Syntax**

Partial plan = \{Steps, Orderings, Aux. Constraints \}

Auxiliary Constraints:

- **Interval preservation constraint (IPC)**
  \( s_1, p, s_2 \)  
  p must be preserved between \( s_1 \) and \( s_2 \)

- **Point truth Constraint (PTC)**
  \( p@s \)  
  p must hold in the state before s
Partial Plans: Semantics

Candidate is any action sequence that
-- contains actions corresponding to all the steps,
-- satisfies all the ordering and auxiliary constraints

P: 0 1: Load(A) 2: Fly() 3: Load(B) 4: Unload(A) In(A)@2

Candidates (∈ «P»)  Non-Candidates (∉ «P»)

[Load(A), Load(B), Fly(), Unload(A)]

Minimal candidate. Corresponds to safe linearization [01324∞]

[Load(A), Load(B), Fly(), Unload(B)]

Corresponds to unsafe linearization [01234∞]

[Load(A), Fly(), Load(B), Unload(B)]

Non-Candidates (∉ «P»)

[Load(A), Fly(), Load(B), Unload(A)]

[Load(A), Fly(), Load(B), Unload(B)]

Deduced candidates

Linearization 1 Linearization 2 Linearization 3 Linearization n

Safe linearization 1 Safe linearization 2 Safe Linearization m

Minimal Cand. 1 + derived candidates

Minimal Cand. 2 + derived candidates

Minimal Cand. m + derived candidates

Refinements

Reduce candidate set size
Increase length of minimal candidates

Linking Syntax and Semantics

Partial Plan

Linearization 1 Linearization 2 Linearization 3 Linearization n

Safe linearization 1 Safe linearization 2 Safe Linearization m

Minimal Cand. 1 + derived candidates

Minimal Cand. 2 + derived candidates

Minimal Cand. m + derived candidates

Refinements

Reduce candidate set size
Increase length of minimal candidates
Refinement (pruning) Strategies

"Canned" inference procedures
-- Prune by propagating the consequences of domain theory and meta-theory of planning onto the partial plan

A refinement strategy \( R : \mathcal{P} \rightarrow \mathcal{P}' \) (\( \mathcal{P}' \) a subset of \( \mathcal{P} \))
- \( R \) is complete if \( \mathcal{P}' \) contains all the solutions of \( \mathcal{P} \)
- \( R \) is monotonic if \( \mathcal{P}' \) has longer minimal candidates than \( \mathcal{P} \)
- \( R \) is progressive if \( \mathcal{P}' \) is a proper subset of \( \mathcal{P} \)
- \( R \) is systematic if components of \( \mathcal{P}' \) don't share candidates

§ A plan set \( \mathcal{P} \) is a set of partial plans \( \{ P_1, P_2 \ldots P_m \} \)

Existing Refinement Strategies

Progression

Regression

PSR

Add in the middle

Plan-Space

State-Space

Extend Prefix

Extend Suffix

At(A,E)

At(B,E)

At(R,E)

Add in the middle
The Refinement Planning Template

Refineplan( P : Plan set)

0*. If « P » is empty, Fail.
1. If a minimal candidate of P is a solution, return it. End
2. Select a refinement strategy R.
   Apply R to P to get a new plan set P'.
3. Call Refine(P')

-- Termination ensured if R is complete and monotonic
-- Solution check done using one of the proofs of correctness

Issues:
1. Representation of plan sets (Conjunctive vs. Disjunctive)
2. Search vs. solution extraction
3. Affinity between refinement and proof used for solution check

A flexible Split&Prune search for refinement planning

Refineplan( P : Plan)

0*. If « P » is empty, Fail.
1. If a minimal candidate of P is a solution, terminate.
2. Select a refinement strategy R.
   Apply R to P to get a new plan set P'.
3. Split P' into k plansets
4. Non-deterministically select one of the plansets P'i
   Call Refine(P'i)
Two classes of refinement planners

<table>
<thead>
<tr>
<th>Conjunctive planners</th>
<th>Disjunctive planners</th>
</tr>
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<tbody>
<tr>
<td>◇ Search in the space of conjunctive partial plans</td>
<td></td>
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<tr>
<td>– Disjunction split into the search space</td>
<td></td>
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<tr>
<td>– search guidance is nontrivial</td>
<td></td>
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<tr>
<td>– Solution extraction is trivial</td>
<td></td>
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<tr>
<td>◇ Examples:</td>
<td></td>
</tr>
<tr>
<td>– STRIPS &amp; Prodigy</td>
<td></td>
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<tr>
<td>– SNLP &amp; UCPOP</td>
<td></td>
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<tr>
<td>– NONLIN &amp; SIPE</td>
<td></td>
</tr>
<tr>
<td>– UNPOP &amp; HSP</td>
<td></td>
</tr>
<tr>
<td>◇ Search in the space of disjunctive partial plans</td>
<td></td>
</tr>
<tr>
<td>◇ Disjunction handled explicitly</td>
<td></td>
</tr>
<tr>
<td>◇ Solution extraction is non-trivial</td>
<td></td>
</tr>
<tr>
<td>▶ CSP/SAT/ILP/BDD methods</td>
<td></td>
</tr>
<tr>
<td>◇ Examples:</td>
<td></td>
</tr>
<tr>
<td>– Graphplan, IPP, STAN</td>
<td></td>
</tr>
<tr>
<td>– SATPLAN</td>
<td></td>
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<tr>
<td>– GP-CSP</td>
<td></td>
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<tr>
<td>– BDDPlan, PropPlan</td>
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</tbody>
</table>

CONJUNCTIVE REFINEMENT PLANNING

Part II
Forward State-space Refinement

- Grow plan prefix by adding applicable actions
  - Complete, Monotonic
    » *consideration of all executable prefixes*
  - Progressive
    » *elimination of unexecutable prefixes*
  - Systematic
    » *each component has a different prefix*
  - Completely specified state
    » Easier to control?
  - Higher branching factor

![Diagram showing Forward State-space Refinement]

Backward State-space Refinement

- Grow plan suffix by adding relevant actions
  - Complete, Monotonic
    » *consideration of all relevant suffixes*
  - Progressive
    » *elimination of irrelevant suffixes*
  - Systematic
    » *each component has a different suffix*
  - Goal directed
    » Lower branching
  - Partial state
    » Harder to detect inconsistency

![Diagram showing Backward State-space Refinement]
Plan-space Refinement

Goal selection:
Select a precondition

Establishment:
Select a step (new or existing) and make it give the condition

De-clobbering:
Force intervening steps to preserve the condition

Book-keeping: (Optional)
Add IPCs to preserve the establishment
⇒ Systematicity

(PSR is complete, and progressive)

(Sacerdoti, 1972; Pednault, 1988; McAllester, 1991)

Tradeoffs among Refinements

FSR and BSR must commit to both position and relevance of actions

+ Give state information (Easier plan validation)
- Leads to premature commitment
- Too many states when actions have durations

Plan-space refinement (PSR) avoids constraining position

+ Reduces commitment (large candidate set /branch)
- Increases plan-validation costs
+ Easily extendible to actions with duration

The Task: great!
Loss Filling holy wars
Issues in instantiating Refineplan

- Although a planner can use multiple different refinements, most implementations stick to a single refinement.
- Although refinement can be used along with any type of correctness check, there is affinity between specific refinements and proof techniques (support finite differencing)
  - FSR and Progression based proof
  - BSR and Regression based proof
  - PSR and Causal proof
- Although it is enough to check if any one of the safe linearizations are solutions, most planners refine a partial plan until all its linearizations are safe
  - Tractability refinements (pre-order, pre-satisfy)

With these changes, an instantiated and simplified planner may “look” considerably different from the general template.

Generating Conjunctive Refinement Planners

Refineplan (P : Plan)

0. If «P» is empty, Fail.
1. If a minimal candidate of «P» is a solution, terminate.
2. Select a refinement strategy R.
   - Apply R to «P» to get a set of new plans P1...Pj
   - Add all plans to the search queue.
3. Non-deterministically select one of the plans P_i
   - Call Refine(P_i)
Case Study: UCPOP

Refineplan (\( \varphi \): Plan)

0*. If \( \mathcal{P} \) is order inconsistent, FAIL.
1. If no open conditions and no unsafe IPCs, SUCCESS.
2. Generate new plans using either 2' or 2''
   Add the plans to the search queue
2'. Remove an open condition \( c@s \) in \( \mathcal{P} \).
   2.1. For each step \( s' \) in \( \mathcal{P} \) that gives \( c \), make a new plan
       \( \mathcal{P}' = \mathcal{P} + (s' < s) + \text{IPC } s' - c - s \)
   2.2. For each action \( A \) in the domain that gives \( c \), make a new plan
        \( \mathcal{P}' = \mathcal{P} + s' + \text{IPC } s' - c - s \)
        2.2.1. For each precondition \( c' \) of \( A \), add
                \( c'@sn \) to the list of open conditions of \( \mathcal{P}' \).
        2.2.2. For each IPC \( s' - p - s'' \), if \( sn \) deletes \( p \), add
                \( [s' - p - s''; sn] \) to the list of unsafe IPCs of \( \mathcal{P}' \).
2''. Remove an unsafe IPC \( [s' - p - s''; s'''] \) from \( \mathcal{P} \).
    Make two plans: \( \mathcal{P}' = \mathcal{P} + s''' < s' \)
    \( \mathcal{P}'' = \mathcal{P} + s'' < s''' \)
3. Non-deterministically select one of the plans \( \mathcal{P} \), from the search queue and Call Refine(\( \varphi \)).

Many variations on the theme..

<table>
<thead>
<tr>
<th>Planner</th>
<th>Termination</th>
<th>Goal Sel.</th>
<th>Bookkeeping</th>
<th>Tractability refinements</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWEAK</td>
<td>MTC</td>
<td>MTC</td>
<td>-none-</td>
<td>-none-</td>
</tr>
<tr>
<td>UA</td>
<td>MTC</td>
<td>MTC</td>
<td>-none-</td>
<td>pre-order</td>
</tr>
<tr>
<td>SNLP / UCPOP</td>
<td>Causal proof</td>
<td>arbitrary</td>
<td>Contrib. Prot.</td>
<td>pre-satisfaction</td>
</tr>
<tr>
<td>TOCL</td>
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</tr>
<tr>
<td>McNonlin Pedestal</td>
<td>Causal proof</td>
<td>arbitrary</td>
<td>Interval Prot.</td>
<td>pre-satisfaction</td>
</tr>
<tr>
<td>SNLP-UA</td>
<td>MTC</td>
<td>MTC</td>
<td>Contrib. Prot.</td>
<td>unambig. ord.</td>
</tr>
</tbody>
</table>

(Kambhampati, Knoblock & Yang, 1995)
Some implemented conjunctive planners

<table>
<thead>
<tr>
<th>Planners</th>
<th>Refinement</th>
<th>Heuristics</th>
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<tr>
<td>UCPOP, SNLP</td>
<td>Plan-space</td>
<td>Fail-first Flaw selection</td>
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<tr>
<td>[Weld et. al.]</td>
<td></td>
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<tr>
<td>UNPOP [McDermott]</td>
<td>Forward state space</td>
<td>Distance heuristics</td>
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<tr>
<td>HSP [Geffner &amp; Bonet]</td>
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<td>FF [Hoffman]</td>
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<tr>
<td>IxTET [Ghallab et al]</td>
<td></td>
<td>Hand-coded search control</td>
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<tr>
<td>RAX [Muscettola et al]</td>
<td></td>
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<tr>
<td>Prodigy</td>
<td>Forward state space</td>
<td>Means-ends analysis</td>
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<tr>
<td>[Carbonell et. al.]</td>
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Conjunctive planners: The State of the Art

- *Vanilla* state-space (FSR/BSR) planners were known to be less efficient than vanilla plan-space planners
  - Several research efforts concentrated on extending plan-space approaches to non-classical scenarios
- However, State-space planners seem to have better heuristic support than plan-space planners at this time
  - Distance-based heuristics seem particularly useful for state-space planners
- But, plan-space planners are alleged to provide better support for incremental planning, durative actions etc.
  - replanning, reuse and plan modification, temporal reasoning
  » IXTET, HSTS, RAX....

Charge: Either develop effective approaches for replanning etc. in state-space planners or develop good heuristics for plan-space planners
Disjunctive Planning

- Idea: Consider Partial plans with disjunctive step, ordering, and auxiliary constraints
- Motivation: Provides a lifted search space, avoids re-generating the same failures multiple times (also, rich connections to combinatorial problems)
- Issues:
  - Refining disjunctive plans
    » Graphplan (Blum & Furst, 95)
  - Solution extraction in disjunctive plans
    » Direct combinatorial search
    » Compilation to CSP/SAT/ILP
Disjunctive Representations
--Allow disjunctive step, ordering and auxiliary constraints in partial plans

Disjunctive Plan Refinements (1)

Indirect unioned approach
+ Maximum pruning power
- Exponential refinement cost

INFEASIBLE
Refining Disjunctive plans (2)

Direct naive approach
- Put all actions at all levels
  - Proposition list contains all conditions
    - Trivial to construct
    - Loss of pruning power (progressivity)
      => too many (minimal) candidates
      => Costlier solution extraction

Refining Disjunctive plans (3)

Enforce partial 1-consistency
- Proposition list avoids unsupported conditions
  + Polynomial refinement time
  - Loss of pruning power (progressivity)
Refining Disjunctive plans (4)

- Enforce (partial) 2-consistency
  - Proposition list maintains interactions between pairs of conditions
    - Polynomial refinement time
    - Higher pruning power
    - Better balance between refinement cost and solution extraction cost

Open issues in disjunctive refinement

- Directed partial consistency
  - Mutex propagation is a form of reachability analysis
    - Relevance analysis?
  - Higher levels of directional consistency?
    - Typically not cost effective

- Supporting refinements other than FSR
  - Direct naïve refinements are easy to support; enforcing an appropriate level of consistency is harder
  - Some “relevance” based approaches exist for BSR
    - Inseperability, backward mutex
      - Kambhampati et. al. ECP, 1997
    - Can be used in conjunction with reachability analysis
  - Enforcing effective consistency for PSR is still virgin territory...
Solution Extraction in Disjunctive Plans

- Verify if some minimal candidate of the disjunctive plan is a solution ("Model Finding")
  - Involves (implicit or explicit) use of proof strategies
- Direct methods: Can be solved using special techniques customized to the disjunctive structure
  - Graphplan backward search; Graphplan local search
- Compilation methods: Generate a set of constraints, whose satisfaction ensures that a substructure of the disjunctive structure is a solution. Find a model for the constraints.
  - Constraints will correspond to lines of proof
    - Progression proof, regression proof, causal proof
  - Constraints can be in any standard form
    - e.g. CSP, SAT, ILP, BDD

Graphplan Backward Search
(Direct Search I)

Objective: Find a sub-graph of the plangraph that corresponds to a valid plan.

Method: Start from the goal propositions at the last level
- Select actions to support the goals so that no two are mutex (choice)
- Recurse on the preconditions of the selected actions
  (recursion ends at the initial state)
  (When backtracking over the goals at a level, memoize them)

Optimizations: Adaptation of DVO, FC, EBL, DDB etc... [Kambhampati, JAIR 2000]
**Backward Search**

![Graph Plan Review](image)

**Other Direct Extraction Strategies**

- **Motivation:** No compelling reason for making the search for a valid subgraph backward, or systematic...
- **Alternatives:**
  - Forward Search (dynamic programming) [Parker & Kambhampati; 98,99; Blum & Langford 98]
  - Systematic Undirectional search [Rintanen, 98]
    - Select an action anywhere in the plan-graph for inclusion in the solution; Propagate consequences (adapts normal CSP Search to plan-graph)
  - Local Search [Gerevini et. al., 99]
Compilation to CSP

CSP: Given a set of discrete variables, the domains of the variables, and constraints on the specific values a set of variables can take in combination, find an assignment of values to all the variables which respects all constraints.

Variables: Propositions (in-A-1, in-B-1, ... at-R-E-0 ...)
Domains: Actions supporting that proposition in the plan

Constraints: Mutual exclusion
- \((\text{in-A-1} = \text{load-A-1}) \land (\text{at-R-M-1} = \text{fly-R-1})\) ; etc..

Activation
\(
\text{in-A-1} \neq \# \land \text{in-B-1} \neq \# \) (Goals must have action assignments)
\(
\text{in-A-1} = \text{load-A-1} \Rightarrow \text{at-R-E-0} \neq \# , \text{at-A-E-0} \neq \# \\
\) (subgoal activation constraints)

Compilation to SAT

SAT is CSP with Boolean Variables

Init: At-R-E-0 & At-A-E-0 & At-B-E-0
Goal: In-A-1 & In-B-1

Graph: “cond at k => one of the supporting actions at k-1”
- \(\text{in-A-1} \Rightarrow \text{load-A-1}\)
- \(\text{in-B-1} \Rightarrow \text{load-B-1}\)
- \(\text{at-R-M-1} \Rightarrow \text{fly-R-1}\)
- \(\text{at-R-E-0} \Rightarrow \text{p-at-R-E-1}\)
- \(\text{load-A-1} \Rightarrow \text{at-R-E-0} \land \text{at-A-E-0}\)
- \(\text{load-B-1} \Rightarrow \text{at-R-E-0} \land \text{at-B-E-0}\)
- \(\text{p-at-R-E-1} \Rightarrow \text{at-R-E-0h}\)
- \(\sim \text{in-A-1} \lor \sim \text{at-R-M-1}\)
- \(\sim \text{in-B-1} \lor \sim \text{at-R-M-1}\) “Mutexes”
Compilation to Integer Linear Programming

ILP: Given a set of real valued variables, a linear objective function on the variables, a set of linear inequalities on the variables, and a set of integrality restrictions on the variables. Find the values of the feasible variables for which the objective function attains the maximum value.

- Of integer programming corresponds closely to SAT problem.

- **Motivations**
  - Ability to handle numeric quantities, and do optimization
  - Heuristic value of the LP relaxation of ILP problems

- **Conversion**
  - Convert a SAT/CSP encoding to ILP inequalities
    - E.g. \( X \vee \neg Y \vee Z \Rightarrow x + (1 - y) + z \geq 1 \)
  - Explicitly set up tighter ILP inequalities (Cutting constraints)
    - If \( X, Y, Z \) are pairwise mutex, we can write \( x + y + z \leq 1 \)
      (instead of \( x + y \leq 1 \); \( y + z \leq 1 \); \( z + x \leq 1 \))

Compilation to Binary Decision Diagrams (BDDs)

- **Idea**: Represent disjunctive plans as BDDs and plan extension as BDD operations
  - Proposition list at level \( k \) is an approximation to the set of states reachable in \( k \) steps.
    - The set can be represented compactly as BDDs
    - Plan growth can be modeled as direct manipulations on BDD
      - Operations such as “action projection” need to be modeled as BDD modifications

BDDs support compact representation and direct manipulation of boolean formulae on a finite set of propositions. Popular in CAD community. Standard algorithms for converting boolean formulae into BDDs and for supporting standard boolean operations on them [Bryant et al.]

- Idea: Represent disjunctive plans as BDDs and plan extension as BDD operations
  - Proposition list at level \( k \) is an approximation to the set of states reachable in \( k \) steps.
    - The set can be represented compactly as BDDs
    - Plan growth can be modeled as direct manipulations on BDD
      - Operations such as “action projection” need to be modeled as BDD modifications
Relative Tradeoffs Offered by the various compilation substrates

- CSP encodings support implicit representations
  - More compact encodings [Do & Kambhampati, 2000]
  - Easier integration with Scheduling techniques
- ILP encodings support numeric quantities
  - Seamless integration of numeric resource constraints [Walser & Kautz, 1999]
  - Not competitive with CSP/SAT for problems without numeric constraints
- SAT encodings support axioms in propositional logic form
  - May be more natural to add (for whom ;-)?
- BDDs are very popular in CAD community
  - Commercial interest may spur faster algorithms (which we can use)

CSP Encodings can be more compact: GP-CSP

<table>
<thead>
<tr>
<th>Problem</th>
<th>Graphplan time (s)</th>
<th>Graphplan mem</th>
<th>Satz time(s)</th>
<th>Satz mem</th>
<th>Relsat time (s)</th>
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Do & Kambhampati, 2000
Advantages of CSP encodings over SAT encodings: GP-CSP

<table>
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<tr>
<th>Problems</th>
<th>Size-based EBL</th>
<th>Relevance-based EBL</th>
<th>Size of learning: k = 10 for both size-based and relevance-based</th>
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<td>38.01</td>
<td>86 M</td>
<td>23.89</td>
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</tbody>
</table>

Direct vs. compiled solution extraction

DIRECT

- Need to adapt CSP/SAT techniques
- Can exploit approaches for compacting the plan
- Can make the search incremental across iterations

Compiled

- Can exploit the latest advances in SAT/CSP solvers
- Compilation stage can be time consuming, leads to memory blow-up
- Makes it harder to exploit search from previous iterations
- Makes it easier to add declarative control knowledge
Disjunctive planners based on BSS and PS refinements?

Graphplan can be seen as a disjunctive planner based on state-space refinement.
- How about planners based on other refinements?

Chief difficulty lies in generalizing the refinements to disjunctive plans, while retaining their progressivity
- The mutex propagation step of Graphplan is what makes it progressive
- Generalization is however quite easy if we sacrifice “progressiveness” of refinements

Basic idea:
- Set up bounded length disjunctive structure
- Search for a substructure that satisfies solution properties
  » Use lines of proofs

[Mak & Kambhampati, 1999]

Lines of Proof as basis for (naïve) encodings

- Loop from k=1, until a solution is found
  - Set up k-length sequence of disjunctive actions (a1 V a2 V ....V an)
    » In effect, a direct naïve refinement is used (monotonic, complete, but not progressive)
  - Impose constraints, satisfaction of which ensures that some sub-sequence of the disjunctive sequence is a solution
    » The constraints set up lines of proof
      - State-space proofs
        - Graphplan’s backward search can be thought of as setting up backward proof
      - Causal proofs
        - Convert the constraints into your favorite combinatorial substrate and solve
          a1    a1    a1    a1
          a2    a2    a2    a2
          a3    a3    a3    a3
          ....  ....  ....  ....
          an    an    an    an
          1     2     3     k
Encodings based on state-based proofs

Propositions corresponding to initial conditions and goals are true at their respective levels:
- $P1^0 \& P2^0 \& P7^k \& P9^k$
- At least one of the actions at each level will occur:
  - $a_1 \& a_2 \& \ldots \& a_n$
- Actions imply their preconditions and effects:
  - $a_{i-k} \Rightarrow prec_{i,k-1} \& Eff_{i,k}$

A proposition $P$ remains true if no action occurring at $j+1$ deletes $P$:
- $P_i-j \& Ak-(j+1) \Rightarrow P_i-j$

A proposition $P$ changes values between $j$ and $j+1$ only if an action occurs that makes it so:
- $\neg P_i-j \& P_i-(j+1) \Rightarrow al-j \lor am-j \ldots$

Backward proof (explanatory frame)
Forward proof (classical Frame)

Encodings based on Causal proofs

Each step is mapped to exactly one action:
- $S_i = A_1 \lor S_i = A_2 \ldots \lor (S_i = A_l \& S_i = A_j)$

A step inherits the needs, adds and deletes of the action it is mapped to:
- $S_i = A_j => Adds(S_i, Pa) \& Needs(S_i, Pp) \& Deletes(S_i, Pd)$

A step gets needs, adds and deletes only through mapped actions:
- $Adds(S_i, Pa) => S_i = A_j \lor S_i = A_k \ldots$
- $Needs(S_i, Pp) => S_i = A_j \lor S_i = A_k \ldots$

Every need is established by some step:
- $Needs(S_i, Pp) \Rightarrow Estab(S_i, Pp, Si)$

Establishment with causal links:
- $Estab(S_i, Pp, Si) \Rightarrow Link(S_i, Pp, Si)$
- $Link(S_i, Pp, Si) \Rightarrow Adds(S_i, Pp) \& Precedes(S_i, Pp)$

Precedence is irreflexive, asymmetric and transitive...
Alternative causal encodings

Some preceding step adds
\( \text{Estab}(S_1, P_j, S_i) \Rightarrow \text{Adds}(S_1, P_j) \)
& \( \text{Precedes}(S_1, S_j) \)
For every preceding step deleting a needed condition, there is a white-knight that adds it back
\( \text{Needs}(S_j, P_j) \land \text{Deletes}(S_d, P_j) \land \text{Precedes}(S_d, S_j) \)
\( \Rightarrow \) \( \text{Wknight}(S_1, S_j, S_d, P_j) \lor \text{Wknight}(S_2, S_j, S_d, P_j) \lor \ldots \)
\( \text{Wknight}(S_w, S_j, S_d, P_j) \Rightarrow \text{Precedes}(S_w, S_j) \land \text{Precedes}(S_d, S_w) \land \text{Adds}(S_w, P_j) \)

Whiteknight based establishment

Eliminates the need for \( O(k^2) \) causal link variables

Contiguity based precedence

Eliminates the need for \( O(k^2) \) Transitivity clauses

Tradeoffs between encodings based on different proof strategies

- Progression (classical frame) encodings lead to higher number of clauses, and allow only serial plans
  - \( O(\#\text{prop} \times \#\text{actions} \times \#\text{levels}) \)
  - To allow parallel plans, we need to look at frame axioms with sets of actions, increasing the clauses exponentially
- Regression (explanatory frame) encodings reduce clauses, and allow parallel plans
  - \( O(\#\text{prop} \times \#\text{levels}) \)
- Empirical results validate dominance of regression over progression encodings
  - The SAT compilation of Graphplan plan-graph corresponds to a form of regression-proof based encoding
Tradeoffs between encodings based on different proof strategies

- Causal encodings in general have more clauses than state-space encodings
  - $O(\#\text{actions} \times \#\text{actions} \times \#\text{fluents})$ for causal link variables
    - Could be reduced by using white-knight based proofs
  - $O(\#\text{actions} \times \#\text{actions} \times \#\text{actions})$ clauses for partial ordering
    - Could be reduced by using contiguous ordering
  - However, the best causal encodings will still be dominated by the backward state-space encodings [Mali & Kambhampati, 99]
- Paradoxical given the success of partial order planners in conjunctive planning?
  - Not really! We are using causal proof which is typically longer than state-based proofs, and are not using the flexibility of step insertion.
    - Can be helpful in incremental planning & Plan reuse
    - Are helpful in using causal domain specific knowledge (e.g. HTN schemas)

Some implemented disjunctive planners

<table>
<thead>
<tr>
<th>Disjunctive Planner</th>
<th>Refinement</th>
<th>Solution Extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphplan (Blum/Furst)</td>
<td>Direct Partially 2-consistent refinement with FSR</td>
<td>Direct search on the disjunctive plan</td>
</tr>
<tr>
<td>IPP (Koehler)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STAN (Fox/Long)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GP-EBL (Kambhampati)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SATPLAN (Kautz/Selman)</td>
<td>Naïve direct refinement with FSR, BSR, PSR</td>
<td>Compilation to SAT</td>
</tr>
<tr>
<td>Blackbox (Kautz/Selman)</td>
<td>Same as Graphplan</td>
<td></td>
</tr>
<tr>
<td>GP-CSP (Do/Kambhampati)</td>
<td>Same as Graphplan</td>
<td>Compilation to CSP</td>
</tr>
<tr>
<td>IPP Plan (Walser/Kautz)</td>
<td>Naïve direct refinement with FSR</td>
<td>Compilation to ILP</td>
</tr>
<tr>
<td>BDDPlan (Hans-Peter)</td>
<td>Naïve direct refinement with FSR</td>
<td>Compilation to BDDs</td>
</tr>
<tr>
<td>PROPPPlan (Stoerr)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conjunctive vs. Disjunctive planners

- Progress depends on the effectiveness of the heuristics for judging the goodness of a partial plan
- Space consumption can be regulated
- Better fit with mixed-initiative, incremental planning scenarios (?)
- Space consumption is a big issue
  - Creation and storage of disjunctive structures
    » CSPs do better by supporting implicit representations
- Connection to combinatorial substrate
- Better integration with non-propositional reasoners (?)

Hybrid planners: Controlled splitting & search in the space of disjunctive plans

PART 2

HEURISTICS & OPTIMIZATIONS
Distance Heuristics from Relaxed Problems

Problem: Estimating the distance of a (partial) state from the initial (or goal) state

Solution: Relax the problem by assuming that all subgoals are independent (ignore +ve / -ve interactions between actions)
Solve the relaxed problem and use the length of the solution as part of the heuristic

Properties: The heuristic is neither a lower bound (-ve interactions) nor an upper-bound (+ve interactions).
--leads to inoptimal solutions (in terms of plan length)
>>Possible to reduce inoptimality by considering interactions

History: First used in iXTET [Ghallab et. al. 1995]
hinted at by Smith’s Operator Graphs [Smith et. al., 1995]
independently re-discovered in UNPOP [McDermott, 1996]
independently re-re-discovered in HSP [Bonet & Geffner, 1997]

---

Top down greedy distance computation

For each partial plan in the search queue, estimate its h-value using this procedure

---

A. A. Artificial Intelligence/Unrelaxed Problems

---

P1, P2 A1 G1
Q11 B1 P1
Q11 B2 P2,~P1
Q13 C2 P2
Q12 D1 Q11

Init: Q12 Goal: G1
Bottom-up Distance computation

- Each condition (state-variable & value combo) has a distance
  - Initialized to 0 for all conditions in the initial state
  - $\infty$ for everything else
- Repeat until reaching fix-point (until no distances change)
  - Select an action $A$ to be applied. Let $P_1 \ldots P_m$ be its preconditions and $E_1 \ldots E_n$ be the conditions it adds.
  - Reset the distance values of $E_i$ as follows

\[
\text{Dist}(E_i) = \min \left( \text{Dist}(E_i), \text{Dist}(\{P_1, \ldots, P_i\}) + 1 \right)
\]

\[
\text{Dist}(\{P_1, \ldots, P_i\}) = \sum_i \text{Dist}(P_i)
\]

Improving the Heuristic

- Subgoal independence assumption leads to
  - Inadmissibility (when subgoals interact positively; $A$ is +ve)
  - Un-informedness (when subgoals interact negatively; $I$ is large)
- Informedness improved by accounting for -ve interactions
  - Use information about infeasible subgoal sets (“Mutexes”)
    - No plan of length $k$ can achieve $p$ and $q$
    - No plan can ever achieve $p$ and $q$
- Admissibility can be improved by accounting for +ve interactions
  - Solve the planning problems ignoring -ve interactions between actions to get lower bound on plan length (distance)
Using the Planning Graph to account for +ve/-ve Interactions

- Mutex relations can help account for -ve interactions
  - Grow the PG until level-off (Polynomial operation)
  - Distance of a set \( \{P_1, P_2, \ldots, P_n\} \) is the index of the first level of the PG where \( P_1 \ldots P_n \) are present without any mutexes
    - Easy to generalize to n-ary mutexes
    - Informedness can be improved by partitioning \( P_1 \ldots P_n \) into some \( k \) partitions, \( S_1 \ldots S_k \) and summing the distances of \( S_i \)

- +ve interactions can be accounted for by extracting a plan from the PG ignoring mutex relations
  - Run Graphplan algorithm, but do not do any mutex propagation.

Insight: Ideas of disjunctive planning help in generating heuristics for conjunctive planners!

Plan Graph produces a large spectrum of effective heuristics with differing tradeoffs

<table>
<thead>
<tr>
<th>Problem</th>
<th>Graphplan</th>
<th>Supergen</th>
<th>as-yet</th>
<th>pattern-1</th>
<th>pattern-2</th>
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</tbody>
</table>

Table 2: Number of ad-hoc'ed Total CPU Time in Seconds. The dash (-) indicates that no solution was found in 5 hours or 10MPS.
Graphplan
Plan extension phase (based on STAN)

Planning Graph

Extraction of Heuristics

Actions in the Last Level

Regression Planner (based on HSP-R)

Solution Plan

Problem Spec (Init, Goal state)

Action Templates

AltAlt shows quite impressive performance

AltAlt shows quite impressive performance
The cost of heuristic computation can be controlled...

Distance based heuristics help CSP Search too!

- Variable ordering: Pick the goal with the highest level first
- Value ordering: Pick the action whose precondition literals appear unmuted at the lowest level.

Can give huge speedups
(5000x on rocket-world, 100x on gripper problems)
Is insensitive to the length of the planning graph
   -- Can start search in planning graphs of lengths higher than the minimal solution bearing planning graph
   -- Rocket-exthb goes from 29 to 32 steps and 0.03 to 0.01 sec
   -- Avoids exhaustive search in failing levels
Using Distance Heuristics

- Conjunctive FSS planners need to recompute the heuristic as the init state changes
  - UNPOP [McDermott], HSP [Bonet & Geffner], FF [Hoffman]
  - GRT [Refanidis & Vlahavas] elaborates the goal state and computes heuristic in the reverse direction
- Conjunctive BSS planners can get by with a single computation of the heuristic
  - HSP-R [Bonet & Geffner], AltAlt [Long & Kambhampati &...]
- Conjunctive PS planners can use distance heuristics and mutex information to rank partial plans and flaw resolution choices
  - IxTET [Ghallab et. al.]; AltAlt-UC [Long & Kambhampati]
- Disjunctive planners can use distance heuristics as the basis for variable and value ordering heuristics
  - GP-HSP [Kambhampati & Nigenda], GP-CSP [Do & Kambhampati]

Handling Resources, Metric and Temporal Constraints
Adapting to Metric/Temporal Planning

Schedulers already routinely handle resources and metric/temporal constraints.
- Let the “planner” concentrate on causal reasoning
- Let the “scheduler” concentrate on resource allocation, sequencing and numeric constraints for the generated causal plan

Need better coupling to avoid inter-module thrashing....

Issues in handling time and resources

- Monolithic or loosely-coupled?
- How can the Planner keep track of consistency?
  - Low level constraint propagation
    » Loose path consistency on TCSPs
    » Bounds on resource consumption,
    » LP relaxations of metric constraints
  - Pre-emptive conflict resolution
    The more aggressive you do this, the less need for a scheduler...
- How do the modules interact?
  - Failure explanations; Partial results
- Which Planners are best suited for time and resources?**
**RealPlan--Master/Slave**

- Planner does causal reasoning.
- Scheduler attempts resource allocation.
- If scheduler fails, planner has to restart.

### Performance of Graphplan vs Planning-Scheduling in the Mobile Robot Problem in Offline World

![Graph showing performance comparison]

- **RealPlan-MS**
- **RealPlan-PP**

### Problem Table

<table>
<thead>
<tr>
<th>Problem</th>
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<th>RealPlan-MS</th>
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**RealPlan: Peer-to-Peer**

- Explanation-directed backtracking between Planner and Scheduler

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What planners are good for handling resources and time?

- State-space approaches have an edge in terms of ease of monitoring resource usage
  - Time-point based representations are known to be better for multi-capacity resource constraints in scheduling
- Plan-space approaches have an edge in terms of durative actions and continuous change
  - Notion of state not well defined in such cases (Too many states)
  - PCP representations are known to be better for scheduling with single-capacity resources
- Disjunctive compilation approaches may provide better basis for interacting with schedulers and other external constraint reasoners
  - Conjunctive approaches better for monolithic architectures (disjunctive ones may lead to very large encodings)

Tradeoffs in the current implementations

<table>
<thead>
<tr>
<th>Planner</th>
<th>Multi-capacity Resources</th>
<th>Metric Quantities</th>
<th>Optimization</th>
<th>Continuous Time</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphplan</td>
<td>hard?</td>
<td>hard IPP</td>
<td>hard?</td>
<td>hard TGP</td>
<td>good</td>
</tr>
<tr>
<td>SAT Compilation</td>
<td>moderate?</td>
<td>moderate LPSAT</td>
<td>moderate?</td>
<td>very hard?</td>
<td>good</td>
</tr>
<tr>
<td>ILP Compilation</td>
<td>easy?</td>
<td>easy ILP-plan</td>
<td>easy ILP-plan</td>
<td>very hard?</td>
<td>fair?</td>
</tr>
<tr>
<td>Conjunctive PS</td>
<td>moderate IxTeT</td>
<td>moderate Zeno</td>
<td>moderate?</td>
<td>easy HSTS</td>
<td>fair?</td>
</tr>
</tbody>
</table>

Work on loosely-coupled architectures is just starting... [Srivastava, 2000]
PART 3  CUSTOMIZING PLANNERS WITH DOMAIN SPECIFIC KNOWLEDGE

1. User-assisted customization
2. Automated customization

Improving Performance through Customization

✧ Biasing the search with control knowledge acquired from experts
  – Non-primitive actions and reduction schemas
  – Automated synthesis of customized planners
    » Combine formal theory of refinement planning and domain-specific control knowledge

✧ Use of learning techniques
  – Search control rule learning
  – Plan reuse
User-Assisted Customization (using domain-specific Knowledge)

- Domain independent planners tend to miss the regularities in the domain
- Domain specific planners have to be built from scratch for every domain

An “Any-Expertise” Solution: Try adding domain specific control knowledge to the domain-independent planners

Many User-Customizable Planners

- Conjunctive planners
  - HTN planners
    » SIPE [Wilkins, 85-]
    » NONLIN/O-Plan [Tate et. al., 77-]
    » NOAH [Sacerdoti, 75]
    » Also SHOP (Nau et. al., IJCAI-99)
  - State-space planners
    » TLPlan [Bacchus & Kabanza, 95; 99]
    » TALPlan [Kvarnstrom & Doherty, 2000]
  - Customization frameworks
    » CLAY [Srivastava & Kambhampati, 97]
- Disjunctive planners
  - HTN SAT [Mali & Kambhampati, 98]
  - SATPLAN+Dom [Kautz & Selman, 98]
With right domain knowledge any level of performance can be achieved...

- HTN-SAT, SATPLAN+DOM beat SATPLAN...
  - Expect reduction schemas, declarative knowledge about inoptimal plans
- TLPLAN beats SATPLAN, GRAPHPLAN
  - But uses quite detailed domain knowledge
- SHOP beats TLPLAN...(but not TALPlan)
  - Expects user to write a “program” for the domain in its language
    » Explicit instructions on the order in which schemas are considered and concatenated

Types of Guidance

- Declarative knowledge about desirable or undesirable solutions and partial solutions (SATPLAN+DOM; Cutting Planes)
- Declarative knowledge about desirable/undesirable search paths (TLPlan & TALPlan)
- A declarative grammar of desirable solutions (HTN)

Planner specific. Expert needs to understand the specific details of the planner’s search space.

- Procedural knowledge about how the search for the solution should be organized (SHOP)
- Search control rules for guiding choice points in the planner’s search (NASA RAX)
Ways of using the Domain Knowledge

- As search control
  - HTN schemas, TLPlan rules, SHOP procedures
  - **Issues of Efficient Matching**
- To prune unpromising partial solutions
  - HTN schemas, TLPlan rules, SHOP procedures
  - **Issues of maintaining multiple parses**
- As declarative axioms that are used along with other knowledge
  - SATPlan+Domain specific knowledge
  - Cutting Planes (for ILP encodings)
  - **Issues of domain-knowledge driven simplification**
- Folded into the domain-independent algorithm to generate a new domain-customized planner
  - CLAY
  - **Issues of Program synthesis**

Task Decomposition (HTN) Planning

- The OLDEST approach for providing domain-specific knowledge
  - Most of the fielded applications use HTN planning
- Domain model contains non-primitive actions, and schemas for reducing them
- Reduction schemas are given by the designer
  - Can be seen as encoding user-intent
    - **Popularity of HTN approaches a testament of ease with which these schemas are available?**
- Two notions of completeness:
  - Schema completeness
    - (Partial Hierarchicalization)
  - Planner completeness
Modeling Action Reduction

Dual views of HTN planning

- Capturing hierarchical structure of the domain
  - Motivates top-down planning
    - Start with abstract plans, and reduce them
- Many technical headaches
  - Respecting user-intent, maintaining systematicity and minimality
    - Motivates bottom-up planning
    - Ensure that each partial plan being considered is “legal” with respect to the reduction schemas
    - Directly usable with disjunctive planning approaches
  - Phantomization, filters, promiscuity, downward-unlinearizability...
- Capturing expert advice about desirable solutions
  - Motivates bottom-up planning
    - Directly usable with disjunctive planning approaches
- Connection to efficiency is not obvious

Relative advantages are still unclear...

[Barrett, 97]
SAT encodings of HTN planning

- Abstract actions can be seen as disjunctive constraints
  - K-step encoding has each of the steps mapped to a disjunction of the non-primitive tasks
  - If a step s is mapped to a task N, then one of the reductions of N must hold (**The heart of encoding setup**)  
  - The normal constraints of primitive action-based encoding
  - Causal encodings seem to be a natural fit (given the causal dependencies encoded in reduction schemas)

Solving HTN Encodings

Puzzle: How can increasing encoding sizes lead to efficient planning?

Abstract actions and their reductions put restrictions on the amount of step-action disjunction at the primitive level.

Reduction in step-action disjunction propagates
  - e.g. Fewer causal-link variables, Fewer exclusion clauses…

Savings won’t hold if each non-primitive task has MANY reductions
Full procedural control: The SHOP way

Shop provides a "high-level" programming language in which the user can code his/her domain specific planner.

-- Similarities to HTN planning
-- Not declarative (?)

The SHOP engine can be seen as an interpreter for this language.

Blurs the domain-specific/domain-independent divide
How often does one have this level of knowledge about a domain?

Non-HTN Declarative Guidance

Invariants: A truck is at only one location
\( at(\text{truck}, \text{loc1}, I) \) & \( \text{loc1} \neq \text{loc2} \) \( \Rightarrow \) \( \neg at(\text{truck}, \text{loc2}, I) \)

Optimality: Do not return a package to the same location
\( at(\text{pkg}, \text{loc}, I) \) & \( \neg at(\text{pkg}, \text{loc}, I+1) \) & \( I<J \) \( \Rightarrow \) \( \neg at(\text{pkg}, \text{loc}, j) \)

Simplifying: Once a truck is loaded, it should immediately move
\( \neg in(\text{pkg}, \text{truck}, I) \) & \( in(\text{pkg}, \text{truck}, I+1) \) & \( at(\text{truck}, \text{loc}, I+1) \) \( \Rightarrow \) \( \neg at(\text{truck}, \text{loc}, I+2) \)

Once again, additional clauses first increase the encoding size but make them easier to solve after simplification (unit-propagation etc.).
Rules on desirable State Sequences: TLPlan approach

TLPlan [Bacchus & Kabanza, 95/98] controls a forward state-space planner

Rules are written on state sequences using the linear temporal logic (LTL)

LTL is an extension of prop logic with temporal modalities

$\text{U} \quad \text{until}$

$\text{O} \quad \text{next}$

$<> \quad \text{eventually}$

Example:

If you achieve $\text{on}(B,A)$, then preserve it until $\text{On}(C,B)$ is achieved:

$\Box ( \text{on}(B,A) \Rightarrow \text{on}(B,A) \lor \text{on}(C,B) )$

Good towers are those that do not violate any goal conditions

$\text{goodtower}(x) \equiv \text{clear}(x) \land \text{goodtowerbelow}(x)$

$\text{goodtowerbelow}(x) \equiv (\text{ontable}(x) \land \neg \text{GOAL}(\exists y : \text{on}(x, y) \lor \text{holding}(x)) \lor \exists y : \text{on}(x, y) \land \neg \text{GOAL}(\text{clear}(y)) \land \forall z : \text{GOAL}(\text{on}(z, z)) \Rightarrow z = y \land \forall z : \text{GOAL}(\text{on}(z, z)) \Rightarrow z = x \land \text{goodtowerbelow}(y)$

Keep growing “good” towers, and avoid “bad” towers

$\Box (\forall x : \text{clear}(x) \land \text{goodtower}(x) \Rightarrow \Box \text{goodtowerabove}(x) \land \text{badtower}(x) \Rightarrow \Box (\neg \exists y : \text{on}(y, x)) \land (\text{ontable}(x) \land \exists y : \text{GOAL}(\text{on}(x, y)) \land \neg \text{goodtower}(y)) \Rightarrow \Box (\neg \text{holding}(x))$  

The heart of TLPlan is the ability to incrementally and effectively evaluate the truth of LTL formulas.
Folding the Control Knowledge into the planner: CLAY approach

- Control knowledge similar to TLPlan's
- Knowledge is folded using KIDS semi-automated software synthesis tool into a generic refinement planning template
  - Use of program optimizations such as
    » Finite differencing
    » Context-dependent & independent simplification
- Empirical results demonstrate that folding can be better than interpreting rules

Conundrums of user-assisted customization

- Which planners are easier to control?
  - Conjunctive planners are better if you have search control knowledge
    » Forward State Space (according to TLPlan)
    » Plan-space planners (according to HTN approaches)
  - Disjunctive planners are better if your knowledge can be posed as additional constraints on the valid plans
    » Which SAT encoding?
      - HTN knowledge is easier to add on top of causal encodings
- Which approach provides the best language for expressing domain knowledge for the lay user?
  - (Mine--no, no, Mine!)
- What type of domain knowledge is easier to validate?
  - When does it become “cheating”/“wishful-thinking”
    » Foolish not to be able to use available knowledge
    » Wishful to expect deep procedural knowledge...
Automated Customization of Planners

- Domain pre-processing
  - Invariant detection; Relevance detection;
    Choice elimination, Type analysis
    » STAN/TIM, DISCOPLAN etc.
    » RIFO; ONLP
- Abstraction
  » ALPINE; ABSTRIPS, STAN/TIM etc.
- Learning Search Control rules
  » UCPOP+EBL,
    » PRODIGY+EBL, (Graphplan+EBL)
- Case-based planning (plan reuse)
  » DerSNLP, Prodigy/Analogy

Symmetry & Invariant Detection

- Generate potential invariants and test them
  » DISCOPLAN [Gerevini et. al.]
    » Allows detection of higher-order mutexes
  » Rintanen’s planner
    » Uses model-verification techniques
  » STAN/TIM
    » Type analysis of the domain is used to generate invariants
  » ONLP (Peot & Smith)
    » Use operator graph analysis to eliminate non-viable choices
Abstraction

- **Idea**
  - Abstract some details of the problem or actions.
  - Solve the abstracted version.
  - Extend the solution to the detailed version

- **Precondition Abstraction**
  - Work on satisfying *important* preconditions first
    - Importance judged by:
      - Length of plans for subgoals [ABSTRIPS, PABLO]
      - Inter-goal relations [ALPINE]
      - Distribution-based [HighPoint]
  - Strong abstractions (with downward refinement property) are rare
  - Effectiveness is planner-dependent
    - Clashes with other heuristics such as “*most constrained first*”

Example: Abstracting Resources

- Most planners trash by addressing planning and scheduling considerations together
  - Eg. Blocks world, with multiple robot hands
- **Idea: Abstract resources away during planning**
  - Plan assuming infinite resources
  - Do a post-planning resource allocation phase
  - Re-plan if needed

(with Biplav Srivastava)
Learning Search Control Rules with EBL

Explain leaf level failures
Regress the explanations to compute interior node failure explanations
Use failure explanations to set up control rules

Problems:
-- Most branches end in depth-limits
  > No analytical explanation
  > Use preference rules?
-- THE Utility problem
  > Learn general rules
  > Keep usage statistics & prune useless rules

Example Rules (Learned)

UCPOP

If Polished(x)@S & ~Initially-True(Polished(x))
Then
  REJECT
  Stepadd(Roll(x),Cylindrical(x)@s)

Prodigy

If (and (current-node node)
  (candidate-goal node
    (shape obj Cyl))
  (candidate-goal node
    (surface-condition obj polished)))
Then: Prefer (shape obj cyl) to
  (surface-condition obj polished)

Pruning rule
Preference rule
Case-based Planning
Macrops, Reuse, Replay

- Structures being reused
  - Opaque vs. Modifiable
  - Solution vs. Solving process (derivation)
- Acquisition of structures to be reused
  - Human given vs. Automatically acquired
- Mechanics of reuse
  - Phased vs. simultaneous
- Costs
  - Storage & Retrieval costs; Solution quality

Case-study: DerSNLP

- Modifiable derivational traces are reused
- Traces are automatically acquired during problem solving
  - Analyze the interactions among the parts of a plan, and store plans for non-interacting subgoals separately
    - Reduces retrieval cost
  - Use of EBL failure analysis to detect interactions
- All relevant trace fragments are retrieved and replayed before the control is given to from-scratch planner
  - Extension failures are traced to individual replayed traces, and their storage indices are modified appropriately
    - Improves retrieval accuracy

(Ihrig & Kambhampati, JAIR 97)
DerSNLP: Results

![Graphs showing Performance with increased Training and % Solvability with increased training.](image)

(JAIR, 97)

Reuse in Disjunctive Planning

- Harder to make a disjunctive planner commit to extending a specific plan first
- Options:
  - Support opaque macros along with primitive actions
    » Increases the size of k-step disjunctive plan
    » But a solution may be found at smaller k
  - Modify the problem/domain specification so the old plan’s constraints will be respected in any solution (Bailotti et. al.)
  - MAX-SAT formulations of reuse problem
    » Constrain the encoding so that certain steps may have smaller step-action mapping and ordering choices
    » Causal encodings provide better support

[with Amol Mali]
Summary

♦ Focus on “Neo-classical” planning
  Refinement planning provides a unifying view
  – Conjunctive Refinement Planners
  – Disjunctive Refinement Planners
    » Refinement
    » Solution Extraction
    • Direct vs. compilation to CSP/SAT

♦ Heuristics/Optimizations
  – Relaxed Problem heuristics
  – Consistency enforcement

♦ Customization of planners
  – User-assisted
  – Automated

♦ Related approaches to non-classical planning

But who is winning???
(Using AIPS-2000 Competition as a benchmark)

♦ 15 planners took part in AIPS-2000 competition
  – Representatives from all types of planners we discussed

♦ Competition focussed on pure classical problems
  – Separate tracks for domain-independent and user-customized planners

♦ Domain independent:
  – FF (Forward state-space planning; Heuristics based on planning graphs, local search)

♦ Customized:
  – TALPlan (control knowledge about good and bad state sequences, applied to a forward state-space planner)

♦ Million Dollar question: Will the line-up be same for neo-classical planning (durative actions, continuous change etc?)
Status

✧ Exciting times...
  – Many approaches with superior scale-up capabilities
    » Broadened views of planning
  – Many influences (CSP; OR; MDP; SCM)
✧ Ripe for serious applications
  – VICAR [JPL]; DeepSpace monitoring [NASA/AMES]
  – Plant monitoring [Ayslett et. al.]
  – Manufacturing Process Planning [Nau et. al.; Kambhampati et. al]
  – Supply chain management/ Logistics
    » Industrial “Planning” does not have to be optimal scheduling of an inoptimal action selection!

Resources

✧ Mailing Lists
  – Planning list digest
    » http://rakaposhi.eas.asu.edu/planning-list-digest
  – U.K. P & S List
    » http://www.salford.ac.uk/planning/home.html
✧ Special Conferences
  – Intl. Conf. on AI Planning & Scheduling
    » http://www.isi.edu/aips (April 2000, Breckenridge, CO)
  – European Conference on Planning
    » http://www.informatik.uni-ulm.de/ki/ecp-99.html
  – Also, AAAI, IJCAI, ECAI, Spring/Fall Symposia
✧ Courses
    » http://rakaposhi.eas.asu.edu/planning-class.html
Adapting to Incompleteness, Uncertainty and Dynamism

**Environment**
- (not Static)
- (not fully Observable)

**Conflicting Goals**
- perception (imperfect)
- action (nondeterministic)

The most efficient approaches to all these ills are still...
Incomplete Information: Some Implemented Approaches

◊ Conjunctive planners
  - CNLP [Peot & Smith; 92] CASSANDRA [Pryor & Collins, 96] add sensing actions to UCPOP; support tree-shaped plans
  - SADL/PUCCINI [Golden & Weld; 96-98] integrates planning and sensing in the context of a UCPOP-like planner

◊ Disjunctive planners
  - CGP [Smith & Weld, 98] supports conformant planning on Graphplan
  - SGP [Weld et. al., 98] supports conditional planning on Graphplan
    » One plan-graph per possible world; Interactions among plangraphs captured through induced mutexes
  - [Rintanen, 99] converts conditional planning to QBF encodings

Dynamic Environments

◊ PROBLEM: The world doesn’t sit still. Blind execution of a “correct” plan may not reach goals

◊ APPROACHES:
  - PLAN/MONITOR/REPLAN: Monitor the execution; when the observed state differs from the expected one, REPLAN
    » Replanning is like reuse except there is added incentive for minimal modification
      ● Easy to support with conjunctive plan-space planners
      - PRIAR [Kambhampati; 92]; DerSNLP [Ihrig & Kambhampati, 97]
      ● Possible to support with disjunctive causal encodings
      - [Mali, 1999]
  - MONITOR/REACT/LEARN:
    » Policy construction (Universal plans)...
  - MODEL OTHER AGENTS CAUSING CHANGE:
    » Collaborative/Distributed planning
Stochastic Actions

- **PROBLEM:** Action effects are stochastic
  - Actions transform *state-distributions* to *state-distributions*
  - Maximize “probability” of goal satisfaction
  - Plan assessment itself is hard

- **APPROACHES:**
  - Conjunctive planners
    - BURIDAN [Hanks et. al., 95] uses UCPOP techniques to put candidate plans together and assesses them
      - *Multiple /redundant supports*
  - Disjunctive planners
    - Pgraphplan [Blum & Langford, 98] modifies Graphplan to support some forms of stochastic planning
      - Forward search; value propagation; Envelope extension
    - Maxplan [Majercik & Littman, 98] uses EMAJSAT encodings to solve stochastic planning problems
      - Chance variables & Choice variables. Equivalence classes of models that have the same values of choice variables. Find the equivalence class with maximum probability mass.

Complex & Conflicting Goals

- **Problems & Solutions:**
  - Goals that have temporal extent (stay alive)
    - UCPOP, TLPlan, TGP [Smith & Weld, 99]
  - Goals that have mutual conflicts (Sky-dive & Stay Alive)
  - Goals that take cost of achievement into account
  - Goals that admit degrees of satisfaction (Get rich)
    - Branch & Bound approaches; MAXSAT approaches
      - Pyrrhus [Williamson & Hanks; 92]

```
Decision Theoretic Approaches:
Model goals in terms of factored reward functions
for Markov Decision Processes
--Can utilize tricks and insights from classical planning
[Boutilier, Hanks, Dean; JAIR 99]
```
(very) Quick overview of CSP/SAT concepts

◊ Constraint Satisfaction Problem (CSP)
  – Given
    » A set of discrete variables
    » Legal domains for each of the variables
    » A set of constraints on values groups of variables can take
  – Find an assignment of values to all the variables so that none of the constraints are violated

◊ SAT Problem = CSP with boolean variables

◊ TCSP = CSP where variables are time points and constraints describe allowed distances

A solution:

\[
x = A, y = B, u = D, v = E, w = D, l = B
\]
Important ideas in solving CSPs

Variable order heuristics:
Pick the most constrained variable
--Smallest domain, connected to most other variables,
causes most unit propagation,
causes most resource contention,
has the most distance etc...

Value ordering heuristics
Pick the least constraining value

Consistency enforcement
k-consistency; adaptive consistency. (pre-processing)
Forward Checking, unit propagation during search (dynamic)

Search/Backtracking
DDB/EBL: Remember and use interior node failure explanations
Randomized search

Temporal Constraint Satisfaction Problem (TCSP)

- Variables correspond to time points
- Constraints restrict allowable distances between time points
- Constraints can be represented as sets of intervals
  - Simple temporal problem (STP)
    - All constraints have at most one interval (TRACTABLE!!!)
- Solution of general TCSP involves enforcing path-consistency
  - Approximations such as loose path consistency can be used to detect inconsistencies

\[ (0 \leq t_1 - t_0 \leq 3) \]
\[ (1 \leq t_2 - t_1 \leq 5) \] \& \[ (7 \leq t_2 - t_1 \leq 9) \]
What makes CSP problems hard?

Assignments to individual variables that seem locally consistent are often globally infeasible, causing costly backtracking.

The difficulty of a CSP/SAT problem depends on:
- Number of variables (propositions)
- Number of constraints (clauses)
- Arity of constraints
- Degree of local consistency

---

Hardness & Local Consistency

- An n-variable CSP problem is said to be k-consistent iff every consistent assignment for (k-1) of the n variables can be extended to include any k-th variable
  - Directional consistency: Assignment to first k-1 variables can be extended to the k-th variable
  - Strongly k-consistent if it is j-consistent for all j from 1 to k
- Higher the level of (strong) consistency of a problem, the lesser the amount of backtracking required to solve the problem
  - A CSP with strong n-consistency can be solved without any backtracking
- We can improve the level of consistency of a problem by explicating implicit constraints
  - Enforcing k-consistency is of $O(n^k)$ complexity
    - Break-even seems to be around $k=2$ (“arc consistency”) or 3 (“path consistency”)
    - Use of directional and partial consistency enforcement techniques