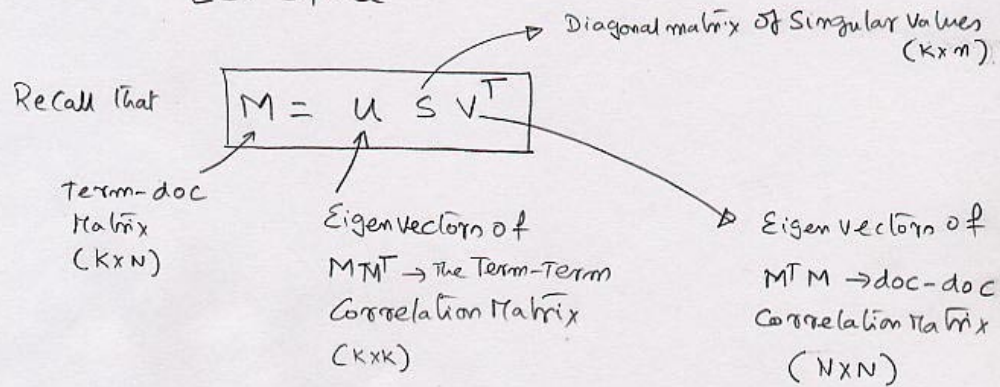


Deriving The Formula for Converting A Query in the normal Space to that in LSI Space



Fact 1: U and V are orthonormal matrices.

That is $U^T U = I$ $V^T V = I$ I is Identity Matrix

makes sense since U and V are made up of Eigen vectors - which are orthogonal unit vectors and thus their dot product will be 1 for the same vectors and zero for non-same pair.

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

If we have the query Q and its representation in LSI Space is D_Q , then Q and D_Q must satisfy the relation

$$Q = U S D_Q^T$$

$$U^T Q = \underbrace{U^T U}_I S D_Q^T \quad - \text{multiply by } U^T \text{ on both sides and use fact 1}$$

$$S^{-1} U^T Q = \underbrace{S^{-1} S}_I D_Q^T \quad - \text{multiply by } S^{-1} \text{ on both sides and by definition } S^{-1} S = I$$

Fact 2: If S is a diagonal matrix S^{-1} will be a diagonal matrix too

$$S^{-1} U^T Q = D_Q^T$$

$$\underbrace{(D_Q^T)^T}_{D_Q} = (S^{-1} U^T Q)^T$$

- Taking Transpose on both sides
noting Transpose of Transpose gives original

By the way Transpose

A bit on Transpose operation

If A is $m \times n$ matrix $[a_{ij}]$.

A^T is $n \times m$ matrix $[a_{ji}]$

(i 'th element of A^T is j 'th element of A)

Fact 3 $(A^T)^T = A$ (easy)

Fact 4 $S^T = S$ If S is a Symmetric Matrix
(ie $s_{ij} = s_{ji}$)

Fact 5 a diagonal matrix is a Symmetric Matrix

$$\begin{bmatrix} \circ & & \\ & \circ & \\ & & \circ \end{bmatrix}$$

Fact 6 $(AB)^T = B^T A^T$

Continuing.

$$D_Q = (S^{-1} U^T Q)^T$$

$$= Q^T (S^{-1} U^T)^T$$

fact 4 with $A = S^{-1} U^T$
 $B = Q$

$$= Q^T \underbrace{(U^T)^T}_U \underbrace{(S^{-1})^T}_{S^{-1} \text{ (using facts 2, 4, 5)}} = \boxed{Q^T U S^{-1}} \quad \text{QED!!}$$