Planning through Stochastic Local Search and Temporal Action Graphs

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Abstract

We present some techniques for planning in domains specified with the recent standard language PDDL2.1, supporting "durative actions" and numerical quantities. These techniques are implemented in LPG, a domain-independent system that took part in the third International Planning Competition (Toulouse 2002). The planner is based on a stochastic local search method and on a graph-based representation called "Temporal Action Graph" (TA-graph). This paper focuses on temporal planning, introducing TA-graphs and proposing some new techniques to guide the search in LPG using this representation. The experimental results of the 3rd IPC, as well as further results presented in this paper, show that our techniques can be very effective, and that often LPG outperforms all other fully-automated planners of the 3rd IPC.

1. Introduction

While in the last two decades most of the research efforts in domain-independent planning concentrated on simple STRIPS domains where actions are instantaneous, modeling temporal and numerical quantities is important for representing real-world domains, where actions take time, consume resources, and the quality of the solutions should take these aspects into account. In the '80s and early '90s several expressive, but inefficient, planning systems handling time have been developed, including (Vere, 1983; Allen, 1991; Penberthy & Weld, 1994; Tsang, 1986). More recently, a number of alternative, interesting approaches to temporal planning has been proposed, e.g., (Dimopoulos & Gerevini, 2002; Smith & Weld, 1999; Do & Kambhampati, 2001; Haslum & Geffner, 2001). Some of these planners can compute plans with optimal makespan, but in practice most of them scale up poorly.

Local search is emerging as a powerful method to address fully-automated planning, though in principle this approach does not guarantee to generate optimal plans. In particular, two planners that successfully participated in the recent 3rd International Planning Competition (IPC) are based on local search: FF (Hoffmann & Nebel, 2001) and LPG. In (Gerevini & Serina, 1999, 2002) we presented a first version of LPG using several techniques for local search in the space of action graphs (A-graphs), particular subgraphs of the planning graph representation (Blum & Furst, 1997). This version handled only STRIPS domains, possibly extended with simple costs associated with the actions. In this paper we present some major improvements that were used in the 3rd IPC to handle domains specified in the recent PDDL2.1 language (Fox & Long, 2001) supporting "durative actions" and numerical quantities.

The general search scheme of our planner is Walkplan, a stochastic local search procedure similar to the well-known Walksat (Selman, Kautz, & Cohen, 1994). Two of the most important extensions on which we focus in this paper concern the use of temporal action graphs (TA-graphs), instead of simple A-graphs, and some new techniques to guide the local search process. In a TA-graph, action nodes are marked with temporal values estimating the earliest time when the corresponding action terminates. Similarly, a fact node is marked with a temporal value estimating the earliest time when the corresponding fact becomes true. A set of ordering constraints is maintained during search to handle mutually exclusive actions, and to represent the temporal constraints implicit in the "causal" relations between actions in the current plan.

The new heuristics exploit some reachability information to weight the elements (TA-graphs) in the search neighborhood that resolve an inconsistency selected from the current TA-graph. The evaluation of these TA-graphs is based on the estimated number of search steps required to reach a solution (a valid plan), its estimated makespan, and its estimated execution cost. LPG is an incremental planner, in the sense that it produces a sequence of valid plans each of which improves the quality of the previous ones.

In the 3rd IPC, our planner showed excellent performance on a large set of test problems in terms of both speed to compute the first solution, and quality of the best solution computed by the incremental process. LPG was the fully-automated planner that solved the highest number of problems, and the one with the highest success ratio between attempted and solve problems.

The paper is organized as follows. Section 2 presents the action and plan representation used in the competition version of LPG. Section 3 describes LPG's local search neighborhood, some new heuristics for temporal action graphs, and the techniques for computing the reachability and temporal information used in these heuristics. Section 4 presents the results of an experimental analysis using the test problems of the 3rd IPC, and illustrating the efficiency of our approach especially for temporal planning. Section 5 gives conclusions and mentions further work. Finally, a collection of appendices describe LPG's algorithm for computing the mutually relations used during search, and give details about some experimental results presented in Section 5.

2. Action and Plan Representation

In this section we introduce our graph-based representations for STRIPS and temporal plans, which can be seen as an elaboration of planning graphs (Blum & Furst, 1997).

A planning graph is a directed acyclic levelled graph with two kinds of nodes and three kinds of edges. The levels alternate between a fact level, containing fact nodes, and an action level containing action nodes. An action node at level t represents an action (instantiated operator) that can be planned at time step t. A fact node represents a proposition corresponding to a precondition of one or more actions at time step t, or to an effect of one or more actions at time step t-1. The fact nodes of level 1 represent the positive facts of the initial state of the planning problem (every fact that is not mentioned in the initial state is assumed false).

In the following, we indicate with [u] the proposition (action) represented by the fact node (action node) u. The edges in a planning graph connect action nodes and fact nodes. In particular, an action node a of level i is connected by: $precondition\ edges$ from the fact nodes of level i representing the preconditions of [a]; add-edges to the fact nodes of level i+1 representing the positive effects of [a]; delete-edges to the fact nodes of level i+1 representing the negative effects of [a]. Each fact node f at a level f is associated with a f no-f action node at the same level, which represents a dummy action having f as its only precondition and effect.

Two action nodes a and b are marked as mutually exclusive in the graph when one of the actions deletes a precondition or add-effect of the other (*interference*), or when a precondition node of a and a precondition node of b are marked as mutually exclusive (*competing needs*).

Two proposition nodes p and q in a proposition level are marked as exclusive if all ways of making proposition [p] true are exclusive with all ways of making [q] true (each action node a having an add-edge to p is marked as exclusive of each action node b having an add-edge to q). When two fact or action nodes are marked as mutually exclusive, we say that there is a mutex relation between them.

Given a planning problem Π , the corresponding planning graph \mathcal{G} can be incrementally constructed level by level starting from level 1 using a polynomial algorithm (Blum & Furst, 1997). The last level of the graph is a propositional level where the goal nodes are present, and there is no mutex relation between them.¹ The mutex relations in the planning graph

^{1.} In some cases, when the problem is not solvable, the algorithm identifies that there is no level satisfying these conditions, and hence it detects that the problem is unsolvable.

monotonically decrease with the increase of the levels: a mutex relation holding at a certain level may not hold at the next levels, but it's guaranteed that it holds at all previous levels. The mutex relations of the last level of the graph are called *global* mutex relations, because they hold at every level of the graph.

Without loss of generality, we can assume that the goal nodes of the last level represent the preconditions of the special action $[a_{end}]$, which is the last action in any valid plan, while the fact nodes of the first level represent the effects of the special action $[a_{start}]$, which is the first action in any valid plan.

Our approach to planning uses particular subgraphs if \mathcal{G} , called *action graphs*, which represent partial plans.

Definition 1 An action graph (A-graph) for \mathcal{G} is a subgraph \mathcal{A} of \mathcal{G} containing a_{end} and such that, if a is an action node of \mathcal{G} in \mathcal{A} , then also the fact nodes of \mathcal{G} corresponding to the preconditions and positive effects of [a] are in \mathcal{A} , together with the edges connecting them to a.

Notice that an action graph can represent an invalid plan for the problem under consideration, since it may contain some *inconsistencies*, i.e., an action with precondition nodes that are not *supported*, or a pair of action nodes involved in a mutex relation. In general, a precondition node q of level i is supported in an action graph \mathcal{A} of \mathcal{G} if either (i) in \mathcal{A} there is an action node at level i-1 representing an action with (positive) effect [q], or (ii) i=1 (i.e., [q] is a proposition of the initial state). An action graph without inconsistencies represents a valid plan and is called *solution graph*.

Definition 2 A solution graph for \mathcal{G} is an action graph \mathcal{A}_s of \mathcal{G} such that all precondition nodes of the actions in \mathcal{A}_s are supported, and there is no mutex relation between action nodes of \mathcal{A}_s .

For large planning problems the construction of the planning graph can be computationally very expensive, especially because of the high number of mutex relations. For this reason our planner considers only pairs of actions that are globally mutex, derived using a dedicated algorithm given in Appendix A. An experimental comparison with the IPP's implementation of the planning graph construction (Koehler, Nebel, Hoffmann, & Dimopoulos, 1997) showed that in practice our method for deriving mutex relations is considerably more efficient than the "traditional" method for deriving the mutex relations in the last level of graph. Moreover, these relations were not more than the relations computed by our method (however, of course, IPP can derive mutex relations at earlier levels than our method cannot compute).

The definition of action graph and the notion of supported fact can be made stronger by observing that the effects of an action node can be automatically propagated to the next levels of the graph through the corresponding no-ops, until there is an interfering action blocking the propagation (if any), or we reach the last level of the graph. The use of the no-op propagation, which we presented in (Gerevini & Serina, 2002), leads to a smaller search space and can be incorporated into the definition of action graph.

Definition 3 An action graph with propagation is an action graph A such that if a is an action node of A at level l, then, for any positive effect [e] of [a] and any level l' > l

of A, the no-op of e at level l' is in A, unless there is another action node at a level l'' ($l \le l'' < l'$) which is mutex with the no-op.

Since in the rest of this paper we consider only action graphs with propagation, we will abbreviate their name simply to action graphs (leaving implicit that they include the no-op propagation).

In most of the existing planners based on planning graphs, when the search for a solution graph fails, \mathcal{G} is iteratively expanded by adding an extra level and performing a new search using the resultant graph. In systematic planners like GRAPHPLAN (Blum & Furst, 1997), STAN (Fox & Long, 1998b) and IPP (Koehler et al., 1997) the search fails when there exists no solution graph, while in planners that use local search like BLACKBOX (Kautz & Selman, 1999) or GPG (Gerevini & Serina, 1999) the search fails when a certain search limit is exceeded. As we will show, in LPG there is no need to explicitly treat this kind of search failure, since the size of the graph is incrementally increased during search (i.e, the graph extension can be part of a search step).

The first version of LPG (Gerevini & Serina, 2002) was based on action graphs where each level may contain an arbitrary number of action nodes, like in the usual definition of planning graph. The newer version of the system that participated in the 3rd IPC uses a restricted class of actions graphs, called *linear action graphs*, combined with some additional data structures supporting a more expressive action and plan representation. In particular, the new system can handle actions having temporal durations and preconditions/effects involving numerical quantities specified in PDDL2.1 (Fox & Long, 2001). In this paper we focus mainly on planning for temporal domains, where LPG showed particularly good performance with respect to the other participants of the 3rd IPC.

In order to keep the presentation simple, we describe our techniques considering mainly preconditions of type "over all" (i.e., preconditions that must hold during the whole action execution) and effects of type "at end" (i.e., effects that hold at the end of the action execution).² In the second part of Section 3.3 we discuss how we handled the other types of preconditions and effects in the test domains of the 3rd IPC.

Definition 4 A linear action graph (LA-graph) of \mathcal{G} is an A-graph of \mathcal{G} in which each level of actions contains at most one action node representing a domain action and any number of no-ops.

It is important to note that having only one action in each level of a LA-graph does not prevent the generation of parallel (partially ordered) plans. In fact, from any LA-graph we can easily extract a partially ordered plan where the ordering constraints are those implicit in the causal structure of the represented plan: an action [a] is ordered before an action [b] only if a has an effect node that is used (possibly through the no-ops) to support a precondition node of b. These causal relations between actions producing an effect and actions consuming it are similar to the causal links in partial-order planning, e.g., (McAllester & D., 1991; Penberthy & Weld, 1992; Nguyen & Kambhampati, 2001). LPG keeps track of these relationships during search and uses them to derive some heuristic

^{2.} The current version of LPG supports all types of preconditions and effects that can be expressed in PDDL2.1.

information useful for guiding the search (more details on this in the next section), as well as to extract parallel plans from the solution graph in STRIPS domains.

Moreover, for temporal domains where actions have durations and plan quality mainly depends on the makespan, rather than on the number of actions or graph levels, the distinction between one action or more actions per level is scarcely relevant. The order of the graph levels should not imply by itself any ordering between actions (e.g., an action at a certain level could terminate before the end of an action at the next level).

A major advantage of using LA-graphs, instead of the more general class of A-graphs, is that the simple structure of LA-graph supports a faster computation of the heuristic information and data structures used by the local search algorithm presented in the next section. This is partly related to the fact that in LA-graphs the unsupported preconditions are the only type of inconsistencies that the search process needs to handle explicitly. Clearly, under the STRIPS assumption that actions are instantaneous, the fact that each action belongs to a different level of the graph prevents the presence of interference and competing needs in any LA-graph. Mutex relations involving a no-op can be used during search to efficiently determine which supported preconditions become unsupported as a consequence of adding a certain action node to the current LA-graph (because these relations block the no-op propagation of the relevant no-ops.)

In general, a clear disadvantage of LA-graphs with respect to A-graphs is the size of the representation, since the number of levels in a LA-graph can be significantly larger than the number levels in the corresponding A-graph. Hence, for some domains A-graphs might be more suitable than LA-graphs, although in all planning problems that we tested the size of LA-graphs was never a problem.

For PDDL2.1 domains involving durative actions, our planner represents temporal information by an assignment of real values to the action and fact nodes of the LA-graph, and by a set Ω of ordering constraints between action nodes. The value associated with a fact node f represents the (estimated) earliest time at which [f] becomes true, while the value associated with an action node a represents the (estimated) earliest time when the execution of [a] can terminate. These estimates are derived from the duration of the actions in the LA-graph and the ordering constraints between them that are stated in Ω .

Definition 5 A temporal action graph (TA-graph) of \mathcal{G} is a triple $\langle \mathcal{A}, \mathcal{T}, \Omega \rangle$ where

- A is a linear action graph;
- T is an assignment of real values to the fact and action nodes of A;
- Ω is a set of ordering constraints between action nodes of A.

The ordering constraints in a TA-graph are of two types: constraints between actions that are implicitly ordered by the causal structure of the plan (\prec_C -constraints), and constraints that are imposed by the planner to deal with mutually exclusive actions (\prec_E -constraints). $a \prec_C b$ belongs to Ω if and only if a is used to achieve a precondition node of b in A, while $a \prec_E b$ (or $b \prec_E a$) belongs to Ω only if a and b are mutually exclusive in A. In the next section we will discuss how ordering constraints are stated by LPG during the search. Given our assumption on the types of action preconditions and effects in temporal domains, an ordering constraint $a \prec b$ (where " \prec " stands for \prec_C or \prec_E) states that the

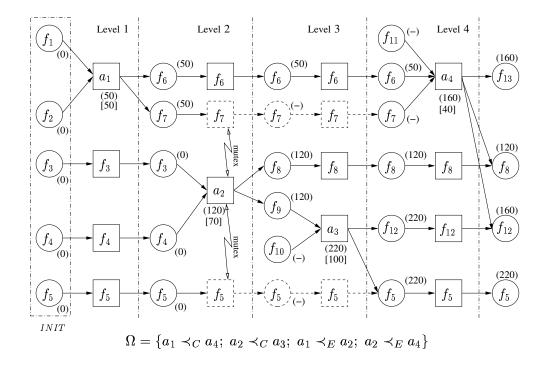


Figure 1: An example of TA-graph. Dashed edges form chains of no-ops that are blocked by mutex actions. Round brackets contain temporal values assigned by \mathcal{T} to the fact nodes (circles) and the action nodes (squares). The numbers in square brackets represent action durations. "(-)" indicates that the corresponding fact node is not supported.

end of [a] is before the start of [b]. The temporal value assigned by \mathcal{T} to a node x will be denoted with Time(x), and it is derived as follows. If a fact node f of the action graph is unsupported, then Time(f) is undefined, otherwise it is the minimum over the temporal values assigned to the actions supporting it. If the temporal value of every precondition nodes of an action node a are undefined, and there is no action node with a temporal value that must precede a according to a, then a is set to the duration of a; otherwise a is the sum of the duration of a and the maximum over the temporal values of its precondition nodes and the temporal values of the actions nodes that must precede a.

Figure 1 gives an example of TA-graph containing four action nodes $(a_{1...4})$ and several fact nodes representing thirteen facts. Since a_1 supports a precondition node of a_4 , $a_1 \prec_C a_4$ belongs to Ω (similarly for $a_2 \prec_C a_3$). $a_1 \prec_E a_2$ belongs to Ω because a_1 and a_2 are mutex (similarly for $a_2 \prec_E a_4$). The temporal value assigned to the facts $f_{1...5}$ at the first level is zero, because they belong to the initial state. a_1 has all its preconditions supported at time zero, and hence $Time(a_1)$ is the duration of a_1 . Since $a_1 \prec a_2 \in \Omega$, $Time(a_2)$ is given by the sum of the duration of a_2 and the maximum over the temporal values of its precondition nodes (zero) and $Time(a_1)$. $Time(a_3)$ is the sum of its duration and the time assigned to f_9 at level 3, which is the only supported precondition node of a_3 . Since f_9 at level 3 is

supported only by a_2 , and this is the only supported precondition node of a_3 , $Time(a_3)$ is the sum of $Time(f_9)$ and the duration of a_3 . Since a_2 must precede a_4 (while there is no ordering constraint between a_2 and a_3), $Time(a_4)$ is the maximum over $Time(a_2)$ and the temporal values of its supported precondition nodes, plus the duration of a_4 . Finally, note that f_{12} at the last level is supported both by a_4 and a_3 . Since $Time(a_3) > Time(a_4)$, we have that $Time(f_{12})$ at this level is equal to $Time(a_4)$.

Definition 6 A temporal solution graph for \mathcal{G} is a TA-graph $\langle \mathcal{A}, \mathcal{T}, \Omega \rangle$ such that \mathcal{A} is a solution LA-graph of \mathcal{G} , \mathcal{T} is consistent with Ω and the duration of the actions in \mathcal{A} , Ω is consistent, and for each pair $\langle a, b \rangle$ of mutex actions in \mathcal{A} , either $\Omega \models a \prec b$ or $\Omega \models b \prec a$.

While obviously the levels in a TA-graph do not correspond to real time values, they represent a topological order for the \prec_C -constraints in the TA-graph (i.e., the actions of the TA-graph ordered according to their relative levels form a linear plan satisfying all \prec_C -constraints). This topological sort can be a valid total order for the \prec_E -constraints of the TA-graph as well, provided that these constraints are appropriately stated during search, i.e., that if a and b are exclusive, the planner appropriately imposes either $a \prec_E b$ or $b \prec_E a$. LPG chooses $a \prec_E b$ if the level of a precedes the level of b, $b \prec_E a$ otherwise. Under this assumption on the "direction" in which \prec_E -constraints are imposed, it is easy to see that the levels of a TA-graph correspond to a topological order of the actions in the represented plan satisfying every ordering constraint in Ω .

For planning domains requiring to minimize the plan makespan (like the "Timed", "SimpleTime', "Complex", and some of the "Numeric" and "HardNumeric" domain sets of the 3rd IPC) each element of LPG's search space is a TA-graph. For domains where time is irrelevant (like the "Strips" domain sets of the 3rd IPC) the search space is formed by LA-graphs.³

In the last part of this section we briefly comment the representation of action durations and action costs in LPG. In accordance with PDDL2.1, our planner handles both static durations and dynamic durations, i.e., durations depending on the state in which the action is applied. Static durations are either explicitly given as numbers specified in the field ":duration" of operator description, or they are implicitly specified by an expression involving some static quantities specified in the initial state of the planning problem. An example of implicit static duration is the duration of the Drive actions in the Depots-Time domain of the 3rd IPC: the Drive operator defines their duration as the distance between the source and the destination of the travel (two parameters of the operator), divided by the speed of the driven vehicle (another operator parameter).

Typically, dynamic durations depend on some numeric quantities that may vary from one state to another state reached by the actions in the plan. An example is the energy of a rover in the domain Rovers-Time of the 3rd IPC, where the duration specified in the recharge operator is

(/ (- 80 (energy ?x)) (recharge-rate ?x))).

This expression depends on the current value of energy for the rover ?x and on its static recharging rate (recharge-rate) specified in the initial state. Our planner handles the

^{3.} We use LA-graphs instead of A-graphs because an experimental analysis showed that the techniques for LA-graphs are more powerful than the techniques for A-graphs presented in (Gerevini & Serina, 2002).

dynamic duration of an action by computing and maintaining during search an estimate of the value of the numerical quantities in the state where the action is applied. However, in this paper we will not describe their treatment in detail. Without loss of generality for the techniques presented in the next section, we can assume that action durations are static.

Each action of a plan can be associated with a cost that may affect the plan quality. Like action durations, in general these costs could be either static or dynamic, though the current version of LPG handles only static ones. LPG precomputes the action costs using the plan metric specified in the problem description using the PDDL2.1 field ":metric". For instance, a plan metric used for a problem in the ZenoTravel-Numeric domain of the 3rd IPC is

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(:metric minimize (+ (* 4 (total-time)) (* 5 (total-fuel-used)))),
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i.e., it is the sum of four times the plan makespan and five times the total amount of the fuel used by the actions in the plan. The cost of an action a is derived by evaluating the metric expression before and after the application of a. The difference of the resultant values is the cost of a. The values assigned to the numeric variables of the expression in the evaluation before the application of a are those specified in the initial state, while their values in the evaluation after the application of the action are those determined by the effects of a. Notice that in these evaluations of the metric expression the temporal value total-time is not considered (if present), because the temporal aspect of the plan quality is already taken into account by the action durations. In the previous example, the metric subexpression used to derive the action costs is (* 5 (total-fuel-used))). Thus, for instance, the cost of the ZenoTravel action (fly plane1 city0 city1) in problem pfile1 of the 3rd IPC is 13560 because the effects of this action increase total-fuel-used of the following quantity

(* (distance city0 city1) (slow-burn plane1))) = 678 * 4 = 2712, which increases the metric value of the plan of 5 * 2712 = 13560.

3. Local Search in the Space of Temporal Action Graphs

In this section we present some search techniques used in the version of our planner that took part in the 3rd IPC. We start with a description of the general local search scheme in the space of action graphs. Then we concentrate on temporal action graphs. In order to simplify the notation, instead of using a and [a] to indicate an action node and the action represented by this node respectively, we will use a to indicate both of them (the appropriate interpretation will be clear from the context).

3.1 Basic Search Procedure: Walkplan

Given a planning graph \mathcal{G} , the local search process of LPG starts from an initial A-graph of \mathcal{G} (i.e., a partial plan), and transforms it into a solution graph (i.e., a valid plan) through the iterative application of some graph modifications improving the current partial plan. The two basic modifications consist of an extension of the A-graph to include a new action

^{4.} For simple STRIPS domains, where there is no metric expression to minimize, the cost of each action is set to one, and LPG minimizes the number of actions in the plan.

node, or a reduction of the A-graph to remove an action node (and the relevant edges).⁵ At any step of the search process, which produces a new A-graph, the set of actions that can be added or removed is determined by the inconsistencies that are present in the current A-graph.

The general scheme for searching a solution graph (a final state of the search) consists of two main steps. The first step is an initialization of the search in which we construct an initial A-graph. The second step is a local search process in the space of all A-graphs, starting from the initial A-graph. We can generate an initial A-graph in several ways. Four possibilities that can be performed in polynomial time, and that we have implemented are: an empty A-graph (i.e., containing only the no-ops of the facts in the initial state, and the special action nodes a_{start} and a_{end}); a randomly generated A-graph; an A-graph where all precondition facts are supported, but in which there may be some violated mutex relations; and an A-graph obtained from an existing plan given in input to the process. The last option is particularly useful in the plan optimization phase, as well as for solving plan adaptation problems (Gerevini & Serina, 2000). In the current version of LPG, the default initialization strategy is the empty action graph. Further details on the initialization step can be found in (Gerevini & Serina, 1999, 2000).

Once we have computed an initial A-graph, each basic search step selects an inconsistency in the current A-graph. If this is an unsupported fact node, then in order to resolve (eliminate) it, we can either add an action node that supports it, or we can remove an action node which is connected to that fact node by a precondition edge. If the chosen inconsistency is a mutex relation, then we can remove one of the action nodes of the mutex relation. Note that the elimination of an action node can remove several inconsistencies (e.g., all those corresponding to the unsupported preconditions of the action removed). On the other hand, obviously the addition of an action node can introduce several new inconsistencies. The strategy for selecting the next inconsistency to handle may have an impact on the overall performance. Our planner includes some strategies that we are currently testing. The default strategy that we have used in all experiments presented in Section 4 prefers inconsistencies appearing at the earliest level of the graph.

Given an action graph \mathcal{A} and an inconsistency σ in \mathcal{A} , the neighborhood $N(\sigma, \mathcal{A})$ of σ in \mathcal{A} is the set of A-graphs obtained from \mathcal{A} by applying a graph modification that resolves σ . At each step of the local search scheme, the elements of the neighborhood are weighted according to a function estimating their quality, and an element with the best quality is then chosen as the next possible A-graph (search state). The quality of an A-graph depends on a number of factors, such as the number of inconsistencies and the estimated number of search steps required to resolve them, the overall cost of the actions in the represented plan and its makespan.

In (Gerevini & Serina, 1999) we proposed three general strategies for guiding the local search: Walkplan, Tabuplan and T-Walkplan. In this paper we focus on Walkplan, which is the

^{5.} Another possible modification that is analyzed in (Gerevini & Serina, 2002), but that will not be considered in this paper, is action ordering, i.e., moving forward or backward one of two exclusive action nodes.

^{6.} For simple STRIPS domain the execution cost of the plan is measured in terms of the number of actions (i.e., each action has cost 1), while plan makespan is ignored, or it can be modeled as number of parallel time steps (Gerevini & Serina, 2002).

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Input: A planning problem \Pi, the maximum number of search steps max\_steps,
           the maximum number of search restarts max\_restarts, a noise factor p \ (0 \le p \le 1).
 Output: A solution graph representing a plan solving \Pi or fail.
1. for i \leftarrow 1 to max_restarts do
         \mathcal{A} \leftarrow an initial A-graph derived from the planning graph of \Pi;
^{2}.
3.
         for j \leftarrow 1 to max\_steps do
4.
               if A is a solution graph then
5.
                  return A
6.
               \sigma \leftarrow an inconsistency in \mathcal{A}:
7.
               N(\sigma, \mathcal{A}) \leftarrow \text{neighborhood of } \mathcal{A} \text{ for } \sigma;
8.
               if \exists \mathcal{A}' \in N(\sigma, \mathcal{A}) such that the quality of \mathcal{A}' is no worse than the quality of \mathcal{A}
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then $\mathcal{A} \leftarrow \mathcal{A}'$ (if there is more than one \mathcal{A}' -graphs, choose randomly one)

 $Walkplan(\Pi, max_steps, max_restarts, p)$

else if random < p then

9.

10.

11.

12.

13. return fail.

Figure 2: General scheme of Walkplan with restarts. random is a randomly chosen value between 0 and 1. The quality of an action graph in the neighborhood is measured using an evaluation function estimating the cost of the graph modification used to generate it from the current action graph.

 $\mathcal{A} \leftarrow$ an element of $N(\sigma, \mathcal{A})$ randomly chosen

else $\mathcal{A} \leftarrow \text{best element in } N(\sigma, \mathcal{A})$;

strategy used by LPG in the 3rd IPC, as well as in the experimental tests presented in Section 4. Walkplan is similar to Walksat, a stochastic local search method for solving propositional satisfiability problems (Selman et al., 1994; Kautz & Selman, 1996). In Walkplan the best element in the neighborhood is the A-graph which has the lowest decrease of quality with respect to the current A-graph, i.e., it does not consider possible improvements. Like Walksat, our strategy uses a noise parameter p. Given an A-graph \mathcal{A} and an inconsistency σ , if there is a modification for σ that does not decrease the quality of \mathcal{A} , then this modification is performed, and the resulting A-graph is chosen as the next A-graph; otherwise, with probability p one of the graphs in $N(\sigma, \mathcal{A})$ is chosen randomly, and with probability 1-p the next A-graph is chosen according to the minimum value of the evaluation function. If a solution graph is not reached after a certain number of search steps (max_steps) , the current A-graph and max_steps are reinitialized, and the search is repeated up to an user-defined maximum number of times $(max_restarts)$. Figure 2 gives a formal description of Walkplan with restarts.

In (Gerevini & Serina, 2002) we proposed some heuristic functions for evaluating the search neighborhood of A-graphs with action costs. In the next section we present additional, more powerful heuristic functions for LA-graphs and TA-graphs. These techniques are implemented in the newer version of our planner and were used in the 3rd IPC.

3.2 Neighborhood and Heuristics for Temporal Action Graphs

The search neighborhood for an inconsistency σ in a LA-graphs \mathcal{A} is the set of LA-graphs that can be derived from \mathcal{A} by adding an action node supporting σ , or removing the action with precondition σ (in linear graphs the only type of inconsistencies are unsupported preconditions). An action a supporting σ can be added to \mathcal{A} at any level l preceding the level of σ , and such that the desired effect of a is not blocked before or at the level of σ (assuming that the underlying planning graph contains a at level a). The neighborhood for a contains an action graph for each of these possibilities.

Since at any level of a LA-graph there can be at most one action node (plus any number of no-ops), when we remove an action node from \mathcal{A} , the corresponding action level becomes "empty" (it contains only no-ops). When we add an action node to a level l, if l is not empty, then the LA-graph is extended of one level, all action nodes from l are shifted forward of one level, and the new action is inserted at level l (Figure 8 in section 3.3 gives an example). Moreover, when we remove an action node a from the current LA-graph, we can remove also each action node supporting only the preconditions of a. Similarly, we can remove the actions supporting only the preconditions of other removed action, and so on. While these induced pruning is not necessary, an experimental analysis showed that it tends to produce plans of better quality more quickly.

The elements of the neighborhood are evaluated according to an action evaluation function E estimating the cost of adding $(E(a)^i)$ or removing an action node a $(E(a)^r)$. In general, E consists of three weighted terms evaluating three aspects of the quality of the current plan that are affected by the addition/removal of a:

$$E(a) = \begin{cases} E(a)^i = \alpha \cdot Execution_cost(a)^i + \beta \cdot Temporal_cost(a)^i + \gamma \cdot Search_cost(a)^i \\ E(a)^r = \alpha \cdot Execution_cost(a)^r + \beta \cdot Temporal_cost(a)^r + \gamma \cdot Search_cost(a)^r \end{cases}$$

The first term of E estimates the increase of the plan execution cost ($Execution_cost$), the second estimates the end time of a ($Temporal_cost$), and third estimates the increase of the number of the search steps needed to reach a solution graph ($Search_cost$). The coefficients of these terms are used to normalize them, and to weight their relative importance (more on this in Section 3.4).

In the computation of the terms of E there is an important tradeoff to consider. On one hand, an accurate evaluation of the elements in the search neighborhood could lead to valid plans of good quality within few search steps. On the other hand, the computation of E should be fast "enough", because the neighborhood could contain many elements, and an accurate evaluation of its elements could slow down the search excessively. First we give a brief intuitive illustration of how these terms are evaluated by LPG, and then we give a more detailed, formal description.

Suppose we are evaluating the addition of a at level l of the current action graph A. The three terms of E are heuristically estimated by computing a relaxed plan π_r containing

^{7.} Note that the empty levels can be ignored during the extraction of the plan from the (temporal) solution graph. They could also be removed during search, if the graph becomes too large. Finally, if the LA-graph contains adjacent empty levels, and in order to resolve the selected inconsistency a certain action node can be added at any of these levels, then the corresponding neighborhood contains only one of the resultant graphs.

a minimal set of actions for achieving (1) the unsupported preconditions of a and (2) the set Σ of preconditions of other actions in the A-graph that would become unsupported by adding a (because it would block the no-op propagation currently used to support such preconditions). This plan is relaxed in the sense that it does not consider the delete-effects of the actions. The derivation of π_r takes into account the actions already in the current partial plan (the plan represented by \mathcal{A}). In particular, the actions in the current plan are used to define an initial state for the problem of achieving the preconditions of a and those in Σ . The relaxed subplan for the preconditions of a is computed from the state $INIT_l$ obtained by applying the actions in \mathcal{A} up to level l-1, ordered according to their corresponding levels. Notice that, as we pointed out in the previous section, the levels in a TA-graph correspond to a total order of the actions of the represented partial-order plan that is consistent with the ordering constraints in Ω (though, of course, this is not necessarily the only valid total order). The relaxed subplan for achieving Σ is computed from $INIT_l$ modified by the effects of a, and it can reuse actions in the relaxed subplan previously computed for the preconditions of a.

The number of actions in the combined relaxed subplans (π_r) is used to define a heuristic estimate of the additional search cost that would be introduced by the new action a. This estimate takes into account also the number of supported preconditions that would become unsupported by adding the actions in π_r to \mathcal{A} . We indicate these subverted preconditions with Threats(a). Using the causal-link notation of partial-order planners (e.g., (McAllester & D., 1991; Penberthy & Weld, 1992; Nguyen & Kambhampati, 2001)), Threats(a) can be formally defined in the following way

$$Threats(a) = \{ f \mid \text{no-op}(f) \text{ and } a \text{ are mutex, } \exists b, c \in \mathcal{A} \text{ such that } b \xrightarrow{f} c \}.$$

Note that, according to our representation, $b \xrightarrow{f} c$ implies Level(b) < Level(a) < Level(c), where Level(x) denotes the level of x in A.

 $Temporal_cost(a)$ is an estimation of the earliest time when the new action would terminate given the actions in π_r .⁸ $Execution_cost(a)$ is an estimation of the additional execution cost that would be required to satisfy the preconditions of a, and is derived by summing the cost of each action a' in π_r (Cost(a')). The terms of $E(a)^r$ are estimated in a similar way. More formally, E is defined as follows:

$$E(a)^{i} \begin{cases} Execution_cost(a)^{i} = \sum_{a' \in Aset(\mathsf{EvalAdd}(a))} Cost(a') \\ Temporal_cost(a)^{i} = End_time(\mathsf{EvalAdd}(a)) \\ Search_cost(a)^{i} = |Aset(\mathsf{EvalAdd}(a))| + \sum_{a' \in Aset(\mathsf{EvalAdd}(a))} |Threats(a')| \\ \end{cases} \\ E(a)^{r} \begin{cases} Execution_cost(a)^{r} = \sum_{a' \in Aset(\mathsf{EvalDel}(a))} Cost(a') - Cost(a) \\ Temporal_cost(a)^{r} = End_time(\mathsf{EvalDel}(a)) \\ Search_cost(a)^{r} = |Aset(\mathsf{EvalDel}(a))| + \sum_{a' \in Aset(\mathsf{EvalDel}(a))} |Threats(a')|. \end{cases}$$

EvalAdd(a)

Input: An action node a that does not belong to the current TA-graph. Output: A pair formed by a set of actions and a temporal value t.

```
1. INIT_l \leftarrow Supported\_facts(Level(a));

2. Rplan \leftarrow RelaxedPlan(Pre(a), INIT_l, \emptyset);

3. t_1 \leftarrow MAX\{0, MAX\{Time(a') \mid \Omega \models a' \prec a\}\};

4. t_2 \leftarrow MAX\{t_1, End\_time(Rplan)\};

5. A \leftarrow Aset(Rplan) \cup \{a\};

6. Rplan \leftarrow RelaxedPlan(Threats(a), INIT_l - Threats(a), A);

7. \mathbf{return} \ \langle Aset(Rplan), t_2 + Duration(a) \rangle.
```

EvalDel(a)

Input: An action node a that belongs to the current TA-graph.

Output: A pair formed by a set of actions and a temporal value t.

```
1. INIT_l \leftarrow Supported\_facts(Level(a));
2. Rplan \leftarrow RelaxedPlan(Unsup\_facts(a), INIT_l, \emptyset).
3. \mathbf{return} \ Rplan.
```

Figure 3: Algorithms for estimating the search, execution and temporal costs for the insertion (EvalAdd) and removal (EvalDel) of an action node a.

EvalAdd(a) is a function returning two values (see Figure 3): the set of actions in π_r (Aset) and an estimation of the earliest time when the new action a would terminate. Similarly for EvalDel(a), which returns a minimal set of actions required to achieve the preconditions that would become unsupported if a were removed from \mathcal{A} , together with an estimation of the earliest time when all these preconditions would become supported. The relaxed subplans used in EvalAdd(a) and EvalDel(a) are computed by RelaxedPlan, a recursive algorithm formally described in Figure 4. Given a set G of goal facts and an initial state $INIT_l$, RelaxedPlan computes a set of actions forming a relaxed plan (ACTS) for achieving G from $INIT_l$, and a temporal value (t) estimating the earliest time when all facts in G are achieved. In EvalAdd and EvalDel these two values returned by RelaxedPlan are indicated with Aset(Rplan) and $End_time(Rplan)$, respectively.

Supported_facts(l) denotes the set of positive facts that are true after executing the actions at levels preceding l (ordered by their level); Duration(a) the duration of a; Pre(a) the precondition nodes of a; Add(a) the (positive) effect nodes of a; $Num_acts(p,l)$ an estimated minimum number of actions required to reach p from $Supported_facts(l)$ (if p is not reachable, $Num_acts(p,l)$ is set to a negative number). The techniques for computing Num_acts and updating Time are described in the next subsection, where we also propose a method for assigning a temporal value to unsupported fact nodes improving the estimation of the earliest end time for an action with unsupported preconditions.

RelaxedPlan $(G, INIT_l, A)$

Input: A set of goal facts (G), the set of facts that are true after executing the actions of the current TA-graph up to level l $(INIT_l)$, a possibly empty set of actions (A);

Output: A set of actions and a real number estimating a minimal set of actions required to achieve G and the earliest time when all facts in G can be achieved, respectively.

```
\begin{array}{l} t \leftarrow \underset{g \in G \cap INIT_{l}}{MAX} Time(g); \\ G \leftarrow G - INIT_{l}; \ ACTS \leftarrow A; \ F \leftarrow \bigcup_{a \in ACTS} Add(a); \end{array}
2.
       t \leftarrow MAX \left\{ t, \underset{g \in G \cap F}{MAX} T(g) \right\}; for all g \in G such that g \not\in F = \bigcup_{a \in ACTS} Add(a)
3.
4.
              bestaction \leftarrow ARGMIN_{\{a' \in A_g\}} \left\{ \max_{p \in Pre(a') - F} Num\_acts(p, l) + |Threats(a')| \right\};
5.
               Rplan \leftarrow RelaxedPlan(Pre(bestaction), INIT_l, ACTS);
6.
7.
               forall f \in Add(bestaction) - F
8.
                        T(f) \leftarrow End\_time(Rplan) + Duration(bestaction);
9.
               ACTS \leftarrow Aset(Rplan) \cup \{bestaction\};
10.
               t \leftarrow MAX\{t, End\_time(Rplan) + Duration(bestaction)\};
        return \langle ACTS, t \rangle.
```

Figure 4: Algorithm for computing a relaxed plan achieving a set of action preconditions from the state $INIT_l$.

After having computed the state $INIT_l$ using $Supported_facts(l)$, in step 2 EvalAdd uses RelaxedPlan to compute a relaxed subplan (Rplan) for achieving the preconditions of the new action a from $INIT_l$. Steps 3–4 compute an estimation of the earliest time when a can be executed as the maximum of the end times of all actions preceding a in \mathcal{A} (t_1) and $End_time(Rplan)$ (t_2) . Steps 5–6 compute a relaxed plan for Threats(a) taking into account a and the actions in the first relaxed subplan.

EvalDel is simpler than EvalAdd, because the only new inconsistencies that can be generated by removing a are the precondition nodes supported by a (possibly through the no-op propagation of its effects) that would become unsupported. $Unsup_facts(a)$ denotes the set of these nodes.

The pair $\langle ACTS, t \rangle$ returned by RelaxedPlan is derived by computing a relaxed plan (Rplan) for G starting from a possibly non-empty input set of actions A that can be "reused" to achieve an action precondition or goal of the relaxed problem. A is not empty whenever RelaxedPlan is recursively executed, and when it is used to estimate the minimum number of actions required to achieve Threats(a) (step 6 of EvalAdd(a)). RelaxedPlan constructs Rplan through a backward process where the action chosen to achieve a (sub)goal g (bestaction) is an action a' such that: (1) g is an effect of a'; (2) all preconditions of a' are reachable from $INIT_l$; (3) reachability of the preconditions of a' require a minimum number of actions,

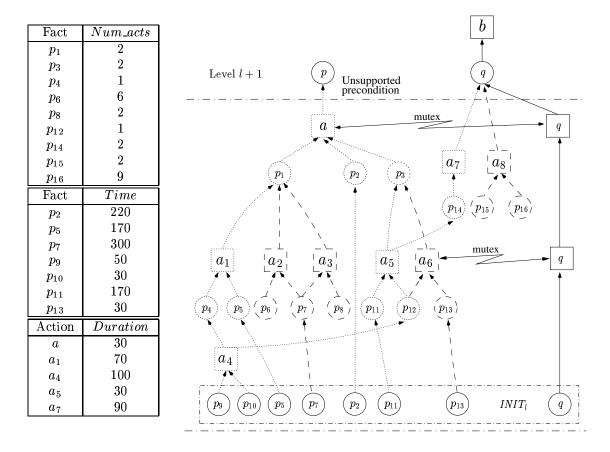


Figure 5: An example illustrating EvalAdd.

estimated as the maximum of the heuristic minimum number of actions required to support each precondition p of a' from $INIT_l$ $(Num_acts(p, l))$; (4) a' subverts the minimum number of supported precondition nodes in \mathcal{A} (Threats(a')). More formally, bestaction is an action satisfying

$$egin{aligned} ARGMIN & \left\{ egin{aligned} MAX \ p \in Pre(a')-F \end{aligned} Num_acts(p,l) + |Threats(a')|
ight\}, \end{aligned}$$

where $A_g = \{a \in \mathcal{O} \mid a \in Add(a), \mathcal{O} \text{ is the set of all actions, } \forall p \in Pre(a) \ Num_acts(p) \geq 0\},$ and F is the set of positive effects of the actions currently in ACTS.

Steps 1, 3 and 7 estimate the earliest time required to achieve all goals in G. This is recursively defined as the maximum of (a) the times assigned to the facts in G that are already true in the state $INIT_l$ (step 1), (b) the estimated earliest time T(g) required to achieve every fact g in G that is an effect of an action currently in ACTS (step 3), and (c) the estimated earliest time required to achieve the preconditions of the action chosen

^{9.} The set \mathcal{O} does not contain operator instances with mutually exclusive preconditions. In principle A_g can be empty because g might not be reachable from $INIT_l$ (i.e., $bestaction = \emptyset$). RelaxedPlan treats this special case by forcing its termination and returning a set of actions including a special action with very high cost, leading E to consider the element of the neighborhood under evaluation a bad possible next search state. For clarity we omit these details from the formal description of the algorithm.

to achieve the remaining facts in G (step 10). The T-times of (b) are computed by steps 7–8 from the relaxed subplan derived to achieve them. Clearly the algorithm terminates, because either every (sub)goal p is reachable from $INIT_l$ (i.e., $Num_acts(p,l) \geq 0$), or at some point $bestaction = \emptyset$ holds, forcing immediate termination (see previous footnote).

Now we illustrate EvalAdd and RelaxedPlan with a worked example (see Figure 5). Suppose we are evaluating the addition of a to the current TA-graph \mathcal{A} . For each fact that is used in the example, the tables of Figure 5 give the relative Num_acts -value or the temporal value (Num_acts for the unsupported facts, Time for the other nodes). The Num_acts -value for a fact belonging to $INIT_l$ is zero. The duration of the actions used in the example are indicated in the corresponding table of Figure 5. Solid nodes represent elements in \mathcal{A} , while dotted and dashed nodes represent actions and preconditions considered during the evaluation process. Dotted nodes indicate the actions and the relative preconditions that are selected by RelaxedPlan.

First we describe the derivation of the action set Aset(Rplan) in steps 2 and 6 of EvalAdd(a), and then the derivation of the temporal values t_1 and t_2 in steps 3–4. Pre(a) is $\{p_1, p_2, p_3\}$ but, since $p_2 \in INIT_l$, in the first execution of RelaxedPlan step 2 removes p_2 from G, and so only p_1 and p_3 are the goals of the relaxed problem. Suppose that in order to achieve p_1 we can use a_1 , a_2 or a_3 (forming the set A_g of step 5). Each of these actions is evaluated by step 5, which assigns a_1 to bestaction. In the recursive call of RelaxedPlan applied to the preconditions of a_1 , p_5 is not considered because it already belongs to $INIT_l$. Regarding the other precondition of a_1 (p_4), suppose that a_4 is the only action achieving it. Then this action is chosen to achieve p_4 , and since its preconditions belong to $INIT_l$, they are not evaluated (the new recursive call of RelaxedPlan returns an empty action set).

Regarding the precondition p_3 of a, assume that it can be achieved only by a_5 and a_6 . These actions have a common precondition (p_{12}) that is an effect of a_4 , an action belonging to ACTS (because already selected by RelaxedPlan $(Pre(a_1), INIT_l, \emptyset)$). The other preconditions of these actions belong to $INIT_l$. Since $|Threats(a_5)| = 0$ and $|Threats(a_6)| = 1$, step 5 of RelaxedPlan selects a_5 . Consequently, at the end of the execution of step 2 in EvalAdd(a) we have $Aset(Rplan) = \{a_1, a_4, a_5\}$.

Concerning the execution of RelaxedPlan for $Threats(a) = \{q\}$ in step 6 of EvalAdd(a), suppose that the only actions for achieving q are a_7 and a_8 . Since the precondition p_{14} of a_7 is an effect of a_5 , which is an action in the input set A (it belongs to the relaxed subplan computed for the preconditions of a), and $Threats(a_7)$ is empty, the best action chosen by RelaxedPlan to support p_{14} is a_7 . It follows that the set of actions returned by RelaxedPlan in step 6 of EvalAdd(a) is $\{a_1, a_4, a_5, a_7\}$.

We now describe the derivation of t_2 in EvalAdd(a), which is an estimation of the earliest start time of a. Consider the execution of RelaxedPlan at step 2. According to the temporal values specified in the table of Figure 5, the value of t at step 1 is $Time(p_2) = 220$. As illustrated above, RelaxedPlan is recursively executed to evaluate the preconditions of a_1 (the action chosen to achieve p_1) and then of a_4 (the action chosen to achieve p_4). In the evaluation of the preconditions of a_4 , at step 1 of RelaxedPlan t is set to 50, i.e., the maximum between $Time(p_9)$ and $Time(p_{10})$, steps 7–8 set $T(p_{12})$ to 50+100 (the duration of a_4), and RelaxedPlan returns $\langle \emptyset, 50 \rangle$. In the evaluation of the preconditions of a_1 , at step 1 of RelaxedPlan t is set to $Time(p_5) = 170$, and at step 10 it is set to $MAX\{170, 50+100\}$. Hence, the recursive execution of RelaxedPlan applied to the preconditions of a_1 returns

 $\langle\{a_4\},170\rangle$, and at step 10 of RelaxedPlan($Pre(a),INIT_l,\emptyset$) t is set to $MAX\{220,170+70\}=240$. The recursive execution of RelaxedPlan applied to the preconditions of a_5 (the action chosen to achieve p_3) returns $\langle\{a_1,a_4,a_5\},200\rangle$. In fact, the only precondition of a_5 that is not in $INIT_l$ (p_{12}) is achieved by an action already in ACTS (a_4). Moreover, since $T(p_{12})=150$ and $Time(p_{11})=170$, the estimated end time of a_5 is 170+30=200. At step 10 of RelaxedPlan($Pre(a),INIT_l,\emptyset$) t is then set to $MAX\{240,200\}$, and the pair assigned to Rplan at step 2 of EvalAdd(a) is $\langle\{a_1,a_4,a_5\},240\rangle$.

At step 3 of EvalAdd(a) suppose that t_1 is set to 230 (i.e., that the highest temporal value assigned to the actions in the TA-graph that must precede a is 230). At step 4 t_2 is set to $MAX\{230, 240\}$, and the execution of RelaxedPlan at step 6 returns $\langle \{a_1, a_4, a_5, a, a_7\}, t_x\rangle$, where t_x is a temporal value that is ignored in the rest of the algorithm, because it does not affect the estimated end time of a. Thus, the output of EvalAdd(a) is $\langle \{a_1, a_4, a_5, a, a_7\}, 240+30\rangle$.

3.3 Computing Reachability and Temporal Information

The techniques described in the previous subsection for computing the action evaluation function use heuristic reachability information about the minimum number of actions required to achieve a fact f from $INIT_l$ $(Num_acts(f,l))$, and earliest times for actions and preconditions. LPG precomputes $Num_acts(f,l)$ for l=1 and any fact f, i.e., it estimates the minimum number of actions required to achieve f from the initial state I of the planning problem before starting the search. For l>1, $Num_acts(f,l)$ can be computed only during search because it depends on which are the actions nodes in the current TA-graph (at levels preceding l). Since during search many action nodes can be added and removed, it is important that the computation of $Num_acts(f,l)$ is fast.

Figure 6 gives ComputeReachabilityInformation, the algorithm used by LPG for computing $Num_acts(f,1)$ trying to take account of the tradeoff between quality of the estimation and computational effort to derive it. The same algorithm could be used for (re)computing $Num_acts(f,l)$ after an action insertion/removal for any l>1 (when l>1, instead of I, in input the algorithm has $Supported_facts(l)$). In addition to $Num_acts(f,1)$, ComputeReachabilityInformation derives heuristic information about the possible earliest time of every fact f reachable from I ($Time_fact(f,1)$). LPG can use $Time_fact(f,1)$ to assign an initial temporal value to any unsupported fact node representing f, instead of leaving it undefined as said in Section 2. This can give a more informative estimation of the earliest start time of an action with unsupported preconditions, which is defined as the maximum of the times assigned to its preconditions.

For clarity we first describe the steps of the algorithm considering only Num_acts , and then we comment the computation of $Time_fact$. In steps 1–4 the algorithm initializes $Num_acts(f,1)$ to 0, if $f \in I$, and to -1 otherwise (indicating that f is not reachable). Then in steps 5–19 it iteratively constructs the set F of facts that are reachable from I, starting with F = I, and terminating when F cannot be further extended. In this forward process each action is applied at most once, and when its preconditions are contained in the

^{10.} In order to obtain better performance, for l > 1 LPG uses an incremental version of ComputeReachabilityInformation updating $Num_acts(f, l)$ after each action insertion/removal. We omit the details of this version of the algorithm.

Compute Reachability Information (I, \mathcal{O})

Input: The initial state of the planning problem under consideration (I) and all ground instances of the operators (\mathcal{O}) ;

Output: An estimate of the number of actions (Num_acts) and of the earliest time $(Time_fact)$ required to achieve each each fact from I.

```
1.
        for all facts f /* the set of all facts is precomputed by the operator instatiation phase */
2.
                 if f \in I then
3.
                      Num\_acts(f,1) \leftarrow 0; Time\_fact(f,1) \leftarrow 0; Action(f) \leftarrow a_{start};
4.
                 else Num\_acts(f,1) \leftarrow -1;
5.
        F \leftarrow I; F_{new} \leftarrow I; A \leftarrow \mathcal{O};
6.
        while F_{new} \neq \emptyset
7.
                 F \leftarrow F \cup F_{new}; F_{new} \leftarrow \emptyset
8.
                 while A' = \{a \in A \mid Pre(a) \subseteq F\} is not empty
9.
                     a \leftarrow \text{an action in } A';
10.
                     ra \leftarrow \mathsf{RequiredActions}(I, Pre(a));
                      t \leftarrow MAX \ Time\_fact(f, 1);
11.
                           f \in Pre(a)
12.
                      forall f \in Add(a)
13.
                               if f \notin F \cup F_{new} or Time\_fact(f,1) > (t + Duration(a)) then
14.
                                      Time\_fact(f,1) \leftarrow t + Duration(a);
                               if f \notin F \cup F_{new} or Num\_acts(f,1) > (ra+1) then
15.
16.
                                      Num\_acts(f,1) \leftarrow ra + 1;
                                      Action(f) \leftarrow a;
17.
                     F_{new} \leftarrow F_{new} \cup Add(a) - I;
18.
19.
                     A \leftarrow A - \{a\};
```

RequiredActions(I, G)

Input: A set of facts I and a set action preconditions G;

Output: An estimate of the minimum number of actions required to achieve all facts in G from I (ACTS).

```
1.
        ACTS \leftarrow \emptyset;
2.
        G \leftarrow G - I;
3.
        while G \neq \emptyset
4.
            g \leftarrow an element of G;
5.
           a \leftarrow Action(q);
6.
            ACTS \leftarrow ACTS \cup \{a\};
7.
            G \leftarrow G \cup Pre(a) - I - \bigcup_{b \in ACTS} Add(b);
8.
        return(|ACTS|).
```

Figure 6: Algorithms for computing heuristic information about the reachability of each fact.

current F. The set A of the available actions is initialized to the set of all possible actions (step 5), and it is reduced after each action application (step 19). The internal loop (steps 8–19) applies the actions in A to the current F, possibly deriving a new set of facts F_{new} in step 18. If F_{new} is not empty, F is extended with F_{new} and the internal loop is repeated. Since F monotonically increases and the number of facts is finite, termination is guaranteed. When an action a in A' (the subset of actions currently in A that are applicable to F) is applied, the reachability information for its effects are revised as follows. First we estimate the minimum number ra of actions required to achieve Pre(a) from I using the subroutine RequiredActions (step 10). Then we use ra to possibly update $Num_acts(f,1)$ for any effect f of a (steps 12–17). If the application of a leads to a lower estimation for f, i.e., if ra+1 is less than the current value of $Num_acts(f,1)$, then $Num_acts(f,1)$ is set to ra+1. In addition, a flag indicating the current best action to achieve f (Action(f)) is set to a.

For any fact f in the initial state, the value of Action(f) is a_{start} (step 3). RequiredActions uses this flag to derive ra through a backward process starting from the input set of action preconditions (G), and ending when $G \subseteq I$. The subroutine incrementally constructs a set of actions (ACTS) achieving the facts in G and the preconditions of the actions already selected (using the flag Action). At each iteration the set G is revised by adding the preconditions of the last action selected, and removing the facts belonging to I or to the effects of actions already selected (step 7). Termination of RequiredActions is guaranteed because every element of G is reachable from I.

 $Time_fact(f,1)$ is computed in a way similar to $Num_acts(f,1)$. Step 3 of ComputeReachabilityInformation initializes it to 0, for any fact f in the initial state. Then, at every application of an action a in the forward process described above, we estimate the earliest possible time t for applying a as the maximum of the times currently assigned to its preconditions (step 11). For any effect f of a that has not been considered yet (i.e., that is not in F), or that has a temporal value higher than t plus the duration of a, $Time_fact(f,1)$ is set to this lower value (because we have found a shorter relaxed plan to achieve f from I).

Figure 7 illustrates the algorithm with an example. Suppose that the facts in the initial state I are $f_{1...8}$, and that the actions in \mathcal{O} are $a_{1...7}$, where the subscript of the actions correspond to the order in which they are applied by the algorithm. The first actions that are applied are a_1 , a_2 and a_3 , because their preconditions are in F, which is initially set to I. The Num_acts -value of these facts is set to zero, because RequiredActions applied to them returns zero. In the internal for-loop of the algorithm we update the reachability information for each effect of these actions. In particular, consider the effects f_1 and f_9 of a_1 . Since f_1 is not a new fact (it belongs to I) and its Num_acts and $Time_fact$ values are set to the minimum (initial) values, steps 14 and 16 do not revise them. Since f_9 is a new fact, step 14 sets $Time_fact(f_9)$ to 10, and step 16 sets $Num_acts(f_9)$ to 1 (ra is zero). Moreover, the flag $Action(f_9)$ is set a_1 . The effects of a_2 and a_3 are handled similarly.

At this point, since there is no other action that is applicable in F, the internal while-loop terminates, F is set to $F \cup \{f_9, f_{10}, f_{11}, f_{12}\}$, and F_{new} is set to \emptyset . The set A' of actions in A that are applicable is $\{a_4, a_5, a_6\}$. Consider the application of a_4 . We have that ra

^{11.} In actual algorithm implemented LPG, when we set Action(f) we consider also the case in which $Num_acts(f,1)$ is equal to ra+1; if the execution cost of a is lower than that cost of the current Action(f), or they have the same cost but the duration of a is lower, then Action(f) is revised to a. For clarity these details are omitted from the formal description of the algorithm.

| Actions | ra | Duration |
|----------|-------------|--------------|
| a_1 | 0 | 10 |
| a_2 | 0 | 30 |
| a_3 | 0 | 50 |
| a_4 | 2 | 50 |
| a_5 | 2 | 70 |
| a_6 | 1 | 30 |
| a_7 | 5 | 20 |
| Facts | Num_acts | $Time_fact$ |
| f_9 | 1 | 10 |
| f_{10} | 1 | 30 |
| f_{11} | 1 | 30 |
| f_{12} | 1 | 50 |
| f_{13} | 3 | 80 |
| f_{14} | 3 | 120 |
| f_{15} | 2 | 80 |
| f_{16} | 2 | 80 |
| f_{17} | 7 | 140 |

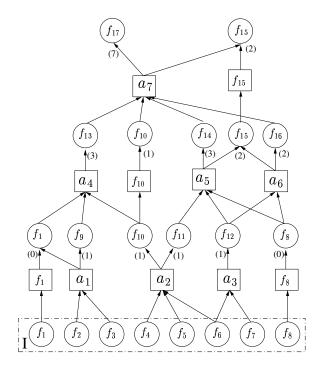


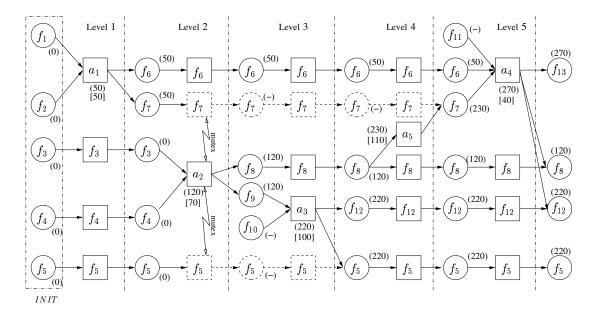
Figure 7: An example illustrating ComputeReachabilityInformation. The numbers in parenthesis are Num_acts -values.

at step 10 is set to 2, because RequiredActions $(I, Pre(a_4))$ sets ACTS to $\{a_1, a_2\}$ (note that $f_1 \in I$, $Action(f_9) = a_1$, $Action(f_{10}) = a_2$, and all preconditions of these actions are in I). Thus, $Time_fact(f_{13})$ is set to 80, and $Num_acts(f_{13})$ to 3. The effects of the actions a_5 and a_6 are handled in a similar way. However, it is worth noting that $Num_acts(f_{15})$ is first set to 3, when we examine a_5 , and then revised to 2, when we examine a_6 . Analogously, $Time_fact(f_{15})$ is first set to 120, and then revised to 80, while $Action(f_{13})$ is first set to a_5 and then to a_6 .

Consider now the preconditions of the last applicable action a_7 . RequiredActions applied to $Pre(a_7)$ returns 6, because the set ACTS of actions selected by the subroutine is $\{a_4, a_1, a_2, a_5, a_3, a_6\}$. Steps 11 of the ComputeReachabilityInformation sets t to $Time_fact(f_{15}) = 120$, and hence the $Time_fact$ -value for the new effect f_{17} is set to 120+20, while its Num_acts -value is set to 6+1.

The complexity of ComputeReachabilityInformation is polynomial in the number of facts and actions in the problem/domain under consideration. Step 10, the most expensive step of the algorithm, is executed $O(\mathcal{O})$ times. It is easy to see that the worst-case time complexity of RequiredActions is $O(\mathcal{O})$. It follows that the time complexity of ComputeReachabilityInformation is $O(\mathcal{O}^2)$. However, we have experimenally observed that very often RequiredActions terminates returning numbers much smaller than $|\mathcal{O}|$ (i.e, that the number of iterations that the algorithm performs is well below $|\mathcal{O}|$).

Concerning the ordering constraints in Ω , if during search the planner adds an action node a to \mathcal{A} for supporting a precondition of another action node b, then $a \prec_C b$ is added



 $\Omega = \{a_1 \prec_C a_4; \ a_2 \prec_C a_3; a_2 \prec_C a_5; a_5 \prec_C a_4; \ a_1 \prec_E a_2; \ a_2 \prec_E a_4\}$

Figure 8: Update of the TA-graph of Figure 1 after the addition of action node a_5 to support the precondition node f_7 of a_4 . Dashed edges form chains of no-ops that are blocked by mutex actions. Round brackets contain temporal values assigned by \mathcal{T} to the fact nodes (circles) and the action nodes (squares). The numbers in square brackets represent action durations. "(-)" indicates that the corresponding fact node is not supported.

to Ω . Moreover, for each action c in \mathcal{A} that is mutex with a, if Level(a) < Level(c), then $a \prec_E c$ is added to Ω , otherwise (Level(c) < Level(a)) $c \prec_E a$ is added to Ω . If the planner removes a from \mathcal{A} , then any ordering constraint involving a is removed from Ω .

The addition/removal of an action node a determines also a possible revision of Time(x) for any fact and action x that is (directly or indirectly) connected to x through the ordering constraints in Ω . Essentially, the algorithm for revising the temporal values assigned to the nodes of \mathcal{A} performs a simple forward propagation starting from the effects of a, and updating level by level the times of the actions (together with the relative precondition and effect nodes) that are constrained by Ω to start after the end of a. When an action node a' is considered for possible temporal revision, Time(a') becomes the maximum temporal values assigned to its precondition nodes plus the duration of a'. The times assigned to the effect nodes of a are revised accordingly. If a' is the only action node supporting an effect f of a, or its temporal value is lower than the value assigned the other action nodes supporting it, then Time(f) is set to Time(a'). For instance, suppose that in order to support the precondition node f_7 of f_7 of f_7 in the TA-graph in Figure 1 we insert the action node f_7 at level 4, which has duration 110 and precondition node f_8 (see Figure 8). Since f_7 is f_7 at level

 $Time(f_7)$ becomes 230, which is propagated to a_4 and its effects. $Time(a_4)$ becomes 270, $Time(f_{12})$ is revised to 220 $(Time(a_3))$, while $Time(f_8)$ remains 120 $(Time(a_2))$.

Some operators in the domains used for the 3rd IPC contain (pre)conditions of type "at end" or effects of type "at start" (Fox & Long, 2001), i.e., preconditions that must hold at the end of the action and effects that are true at the beginning of the action, respectively. In the last part of this section we revise the definition of Time that we have given in the previous section to consider actions involving these types of precondition/effect. When an action node a has a precondition node p of type at end, in the derivation of Time(a) there are two cases to consider, depending on whether p is also an effect node of a or not. In the first case, Time(p) is not considered in the definition of Time(a); in the second case, when we consider the maximum temporal value among all the preconditions of a, instead of using Time(p), we use Time(p) - Duration(a).

If an action node a has an effect node e of type $\mathtt{atstart}$, when we estimate Time(e), instead of using the Time(a) + Duration(a), we use the minimum over (1) Time(a'), for any action node a' supporting e, and (2) Time(a) + Duration(a) (because e is supported by a at the start time of this action). Furthermore, if e is also a precondition node of e of type over all, then Time(e) is not considered in the derivation of Time(a) because the action itself makes this precondition true (unless e is also a precondition node of e of type at start).

When preconditions of type different from over all and effects of type different from at end are present in the domain specification, in some cases two exclusive actions can overlap.¹² The version of LPG that took part in the 3rd IPC did not handle these possible overlappings, and any pair of mutex actions was treated by always imposing an ordering constraint between the end of an action and the start of the other one. However, in all domains of the 3rd IPC partial overlapping of mutex actions was not possible, and so this assumption was harmless. Recently, we have extended the treatment of mutex actions, distinguishing various types interferences and competing needs that can be handled by ordering constraints between different endpoints of the involved actions, which permits some cases of "overlapping exclusive actions".

3.4 Incremental Plan Quality

As we have seen our approach can model different criteria of plan quality determined by action costs and action durations. In the current version of LPG the coefficients α , β and γ of the action evaluation function E are used to weight the relative importance of the execution and temporal costs of E, as well as to normalize them with respect to the search cost. Specifically, LPG uses the following function for evaluating the insertion of an action node a (the evaluation function $E(a)^r$ for removing an action node is analogous):

$$E(a)^i = \frac{\mu_E}{max_{ET}} \cdot Execution_cost(a)^i + \frac{\mu_T}{max_{ET}} \cdot Temporal_cost(a)^i + \frac{1}{max_S} \cdot Search_cost(a)^i,$$

^{12.} For example, is f is a precondition at start of a and $\neg f$ is an effect at end of b, although a and b are mutex, a can overlap b (e.g., a can start after the start time of b and terminate before the end time of b). If a is at a level preceding the level of b, the only ordering constraint that should be imposed is that the start of a is before the end of b, and similarly if b is at a level preceding the level of a.

where μ_E and μ_T are non-negative coefficients that weight the relative importance of the execution and temporal costs, respectively. Their values can be set by the user, or they can be automatically derived from the expression defining the plan metrics in the formalization of the problem. The factors $1/max_{ET}$ and $1/max_S$ are used to normalize the terms of E to a value less than or equal to 1. The value of max_{ET} is defined as $\mu_E \cdot max_E + \mu_T \cdot max_T$, where $max_E \ (max_T)$ is the maximum value of the first (second) term of E over all TA-graphs in the neighborhood, multiplied by the number κ of inconsistencies in the current action graph; max_S is defined as the maximum value of $Search_cost$ over all possible action insertions/removals that eliminate the inconsistency under consideration. The role of κ is to decrease the importance of the first two optimization terms when the current plan contains many inconsistencies, and to increase it when the search approaches a valid plan. I.e., $E(a)^i$ can be rewritten as

$$\begin{split} E(a)^i &= \frac{1}{\kappa \cdot (\mu_E \cdot max_E + \mu_T \cdot max_T)} \cdot \left(\mu_E \cdot Execution_cost(a)^i + \mu_T \cdot Temporal_cost(a)^i \right) \\ &+ \frac{1}{max_S} \cdot Search_cost(a)^i. \end{split}$$

Without this normalization the first two terms of E could be much higher than the value of the third term. This would guide the search towards good quality plans without paying sufficient attention to their validity. On the contrary, especially when the current partial plan contains many inconsistencies, we would like that the search give more importance to reduce the search cost, rather than on optimizing the quality of a plan.

Our planner can produce a succession of valid plans where each plan is an improvement of the previous ones in terms of its quality. The first plan generated is used to initialize a new search for a second plan of better quality, and so on. This is a process that incrementally improves the quality of the plans, and that can be stopped at any time to give the best plan computed so far. Each time we start a new search, some inconsistencies are forced in the TA-graph representing the previous plan, and the resultant TA-graph is used to initialize the search. Similarly, during search some random inconsistencies are forced in the current TA-graph when a valid plan that does not improve the plan of the previous search is reached. This is done by choosing a small set R of action nodes that are removed from the action graph together with (1) the action nodes supporting their preconditions and (2) the action nodes with a precondition supported by an action in R. The elements of R are chosen taking account of the values of μ_E and μ_T . If $\mu_E > \mu_T$, we randomly remove action nodes giving higher probability to those representing actions with higher costs, otherwise preference is given to the action nodes having a higher impact on the plan makespan. ¹³

4. Experimental Results

All our techniques are implemented in LPG. The system is written in C and is available at http://prometeo.ing.unibs.it/lpg. In this section we present some experimental results illustrating the efficiency of LPG using the test problems of the 3rd IPC. These problems belong to several domains, most of which have some variants containing different

^{13.} In the version of LPG that took part in the 3rd IPC this second preference was based on an simple estimation of the temporal impact of each action node. We are currently testing a newer version that selects such actions more accurately by using the *critical path* in the graph of the ordering constraints in the TA-graph.

| Planner | Solved | Attempted | Success ratio |
|---------------------|--------|-----------|---------------|
| LPG | 442 | 468 | 94% |
| FF | 237 | 284 | 83% |
| Simplanner | 91 | 122 | 75% |
| Sapa | 80 | 122 | 66% |
| MIPS | 331 | 508 | 65% |
| VHPOP | 122 | 224 | 54% |
| Stella | 50 | 102 | 49% |
| TP4 | 26 | 204 | 13% |
| TPSYS | 14 | 120 | 12% |
| SemSyn | 11 | 144 | 8% |

Table 1: Number of problems attempted and solved by the planners that took part in the 3rd IPC ordered by their success ratio. The data from the planners compared with LPG are from the official web site of the 3rd IPC.

features of PDDL2.1. The variants are named "Strips", "SimpleTime", "Time", "Complex", "Numeric" and "HardNumeric", and are all handled by our planner. For a description of the domains and of the relative variants the reader may see the official web site of the 3rd IPC (www.dur.ac.uk/d.p.long/competition.html).

All tests were conducted on the official machine of the competition, an AMD Athlon(tm) MP 1800+ (1500MHz) with 1 Gbytes of RAM. The results for LPG correspond to median values over five runs of LPG for each problem considered. The CPU-time limit for each run was 5 minutes, after which termination was forced. Notice that the results that we present here are not exactly the same as the official results of the competition, where for lack of time we were not able to run our system a sufficient number of times to obtain meaningful statistical data. However, in general the new results are very similar to those of the competition, with some considerable improvement in Satellite Complex and in the Rovers domains, where many problems could not be solved due to a bug in the parser of our planner that was easily fixed right after the competition.

Overall, the number of problems attempted in the new tests by our planner was 468 (over a total of 508 problems), and the success ratio was 94.4% (the problems attempted by LPG in the competition were 428 and the success ratio 87%). Figure 1 gives these data for every fully-automated planner that took part in the competition. The success ratio of LPG is the highest one over all competing planners.

The version of LPG that we used in the competition is integrated with an alternative search method that can be activated when the local search is not effective. This method is based on the best-first search technique implemented in FF (Hoffmann & Nebel, 2001). The only domain were we used best-first search instead of local search is FreeCells. The 40 problems that were not attempted by our planner are the 20 problems in Settlers Numeric and the 20 problems in Satellite Hardnumeric. The first domain contains operators

^{14.} When the CPU-time limit was exceeded in 3 or 4 runs, instead of the median value, we considered the worst of the remaining, successful runs. This happened in 15 of the 424 problems solved using Walkplan.

with universally quantified effects, which are not handled in the current version of LPG. The plan metrics of the problems in the second domain require to maximize a certain expression, which is another feature of PDDL2.1 that currently LPG does not handle properly (in principle, many of these problems could be solved by the empty plan, but we do not consider this an interesting valid solution).

We ran LPG with the same default settings for every problem attempted (maximum numbers of search steps and restarts for each run, inconsistency selection strategy, and noise factor), that can be modified by the user.¹⁵ The parameters μ_E and μ_T of the action evaluation function were automatically set using the (linear) plan metric specified in the problem formalization. In particular, μ_E was set to 1, while μ_T was set to the coefficient weighting the total-time variable in the expression specifying the plan metric.¹⁶ If no plan metric is specified, then μ_E was set to 0.5 and μ_T to 0.

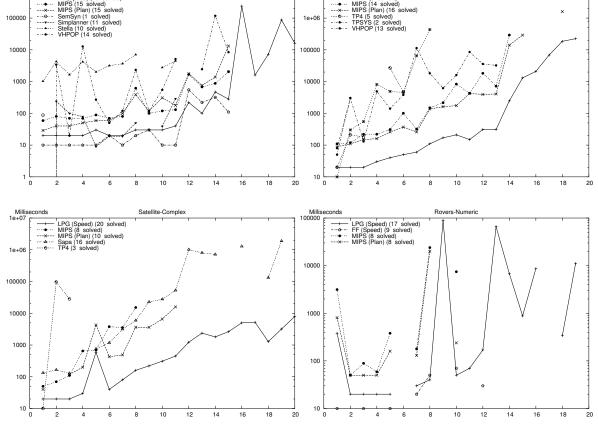
The performance of LPG was tested in terms of both CPU-time required to find a solution (LPG-speed) and quality of the best plan computed (LPG-quality), using at most 5 CPU-minutes. In all plots, on the x-axis we have the problem names (simplified with numbers). On the y-axis, in the plots regarding LPG-speed we have CPU-milliseconds (logarithmic scale), while in the plots regarding LPG-quality we have the quality of the plans computed measured using the plan metric specified for the corresponding problem.

Figure 9 shows the performance of LPG-speed compared to the other competitors in some variants of four domains. Complete results for all other domains and variants are available on-line appendix at http://prometeo.ing.unibs.it/lpg/test-results. In DriveLog Strips FF is on average the fastest planner, but LPG solved more problems, and it scales up somewhat better. In ZenoTravel SimpleTime LPG outperforms the other competitors in terms of both number of problems solved and CPU-time (our planner is about one order of magnitude faster). In Satellite Complex the excellent performance of LPG is even more evident especially for the largest problems. Finally, in Rovers Numeric FF and LPG performs similarly, but our planner solves a larger number of problems. The plots concerning the performance of LPG-quality are given in Appendix B. These results show that the solution computed by our planner was always similar to or better than the solution derived by any other planners. The most interesting differences are in Satellite Complex, where LPG-quality produced solutions of higher quality for almost every problem.

In order to derive some general results about the performance of our approach with respect to all other planners of the competition, we have compared LPG with the best result over all the other planners. We will indicate these results as if they were produced by an hypothetical "SuperPlanner". Clearly, if LPG performs generally better than the SuperPlanner in a certain domain, then in that domain it performs better than any other real planner that we considered. On the other hand, if it performs worse, this does not necessarily imply that there is a single real planner that generally performs better than LPG. The plots of Figures 10 and 11 give complete results for Satellite, one of the domains where

^{15.} The default initial value of the noise p is 0.1. Note that this is a dynamic value that is automatically increased/decreased by the planner during search, depending on the variance of the number of inconsistency in the last n search steps. In all our tests p was automatically increased if the variance did not change significantly in the last 50 search steps; it was set to the initial default value otherwise.

^{16.} For instance, in the example of plan metric given at the end of Section 2, the coefficient weighting total-time is 4, and so for that problem μ_T was set to 4.



ZenoTravel-SimpleTime

DriverLog-Strips

Figure 9: CPU-time and number of problems solved by the planners of the 3rdIPC for the domains DriverLog Strips, ZenoTravel SimpleTime, Satellite Complex, Depots Time and Rovers Numeric.

our planner performed paticularly well in the temporal and Complex variants. The results in these and in the following plots are mostly self explanatory. In the temporal and complex variants LPG-speed is one or more orders of magnitude faster than the SuperPlanner. In the Strips variant the SuperPlanner is faster for the smallest problems, but it is generally slower for the largest ones. the Numeric variant the SuperPlanner is faster, but our planner produces solutions of better quality. In general, regarding LPG-quality, in all variants except Satellite Strips our planner performed always much better than the SuperPlanner. In the Strips variant the quality of the plans produced by our planner are approximately the same as those of the SuperPlanner.

Since our main focus in this paper is temporal planning, it is interesting to compare LPG and the SuperPlanner in the Time variant of all competition domains. The detailed results of this comparison are given in Appendix C. As shown by the plots in this appendix, LPG-speed is usually faster than the SuperPlanner, and it always solves a larger number

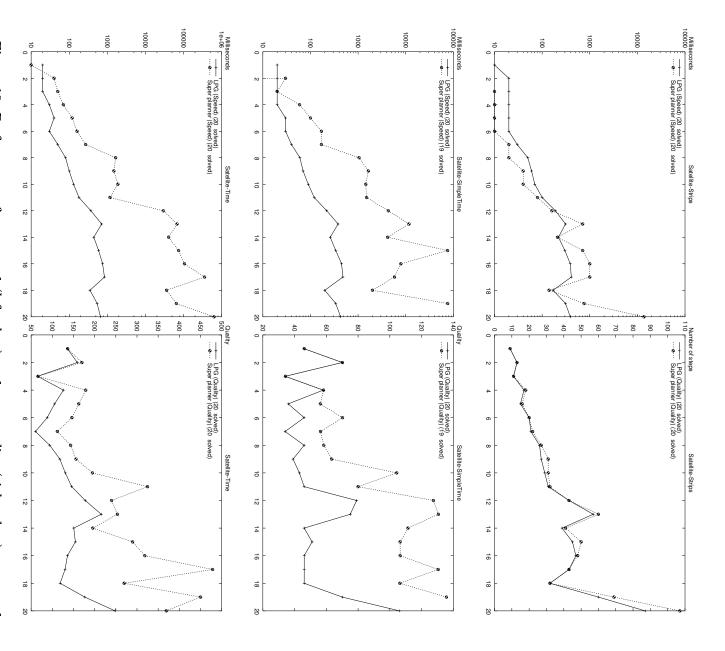


Figure 10: Performance of LPG-speed (left plots) and LPG-quality (right plots) compared with the SuperPlanner in Satellite Strips, SimpleTime and Time.

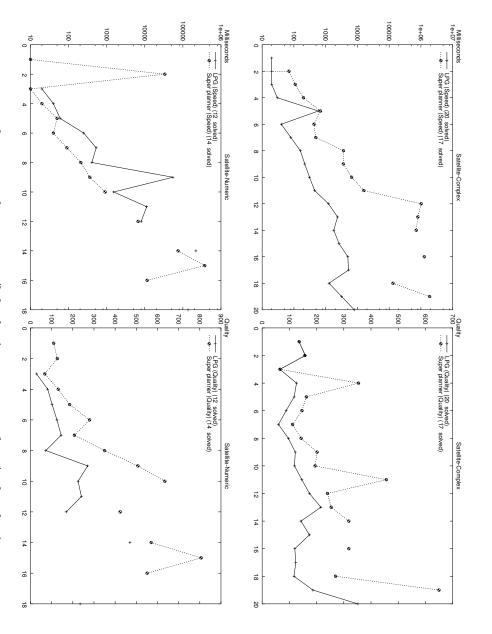


Figure 11: Performance of LPG-speed (left plots) and LPG-quality (right plots) compared with the SuperPlanner in Satellite Complex and Numeric.

problems solved by LPG-speed is 95.1%, while those solved by the SuperPlanner is 77.5%. general performed well in the temporal domains (Edelkamp, 2002). The percentage of the SuperPlanner. This problem was solved by MIPS, another planner of the 3rd IPC that in which it is slower is 13.7%. The percentage of the problems in which our planner is faster is 81.4%, the percentage in of problems, except in ZenoTravel, where our planner solves one problem less than the

in which LPG-speed is faster/slower produces a solution of better/worse quality are the same as the percentages of the problems the Time variant of all the domains the percentages of the problems in which our planner performs better, and in Satellite, where our planner performs always better. Overall, in with some significant differences in ZenoTravel, where in a few problems the SuperPlanner duced by our planner is similar to the quality of the plans generated by the SuperPlanner, Concerning LPG-quality, generally in these domains the quality of the best plans pro-

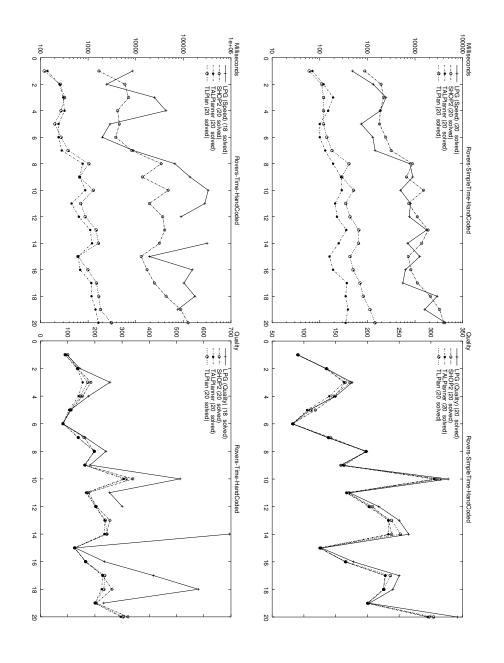


Figure 12: Performance of LPG in two temporal domains designed for hand-coded planners competing at the 3rd IPC.

including the Numeric and Strips variants. SuperPlanner. This is mainly due to the general good performance of FF in these domains. variants. In the Numeric and Strips problems, on average LPG-speed is less efficient than the better than SuperPlanner are even higher than the corresponding percentages for the Time all other domains and problems attempted. Appendices D.1–2 give summary results. However, note that LPG-quality on average is better than the SuperPlanner in every domain, SuperPlanner, and the percentages of problems in which LPG-speed and LPG-quality perform for the Time-problems, in the SimpleTime problems LPG solves more problems than the We have analyzed the performance of LPG with respect to the SuperPlanner also for

71/11.6% of the problems. SuperPlanner in 55.8/38.11% of the problems, while LPG-quality performs better/worse in Overall, considering all problems attempted, LPG-speed performs better/worse than the

research can reduce this gap significantly, but we feel somewhat optimistic about this. this domain LPG-speed performs generally slightly better than SHOP2 (Nau, Munoz-Avila, temporal variants of Rovers. LPG solved 38 of the 40 problems attempted. In terms of plan comparing the performance of LPG and the competing hand-coded planners for the two problems more efficiently than the hand-coded planners. This experiment was aimed at used for testing the hand-coded planners. Of course, we did not expect to solve these III, 500 MHz, with 1 Gbyte of RAM, which is more than two times slower than the machine hand-coded planners in the 3rd IPC. In this experiment LPG was tested using a PC Pentium the problems, but it does not perform so well. It remains an open question whether further Cao, Lotem, & Mitchell, 2001). In Rovers-Time-HandCoded LPG-speed can solve most of by the hand-coded planners, especially in Rovers-SimpleTime-HandCoded. Interestingly, in quality, very often LPG-quality generates plans that are nearly as good as those computed testing how far we are from planners exploiting domain knowledge. Figure 12 shows plots Finally, we ran our planner on some of the large problems that were used to test the

5. Conclusions and Future Work

and effects that can be specified using PDDL2.1. planning competition. Although for simplicity we assumed operator preconditions of type a planner awarded for "distinguished performance of the first order" at the last international over all and effects of type at end, our current planner can handle all types of preconditions We have presented some new techniques for temporal planning that are implemented in LPG,

in the action preconditions and effects, as well as the problem goals. elements and their evaluation can slow down the search excessively; different strategies to in this paper concern: the restriction of the search neighborhood when it contains many choose the inconsistency to handle at each search step; the treatment of numerical quantities Further related techniques that are implemented in LPG, but that we have not described

expression defining the plan quality. the term representing the search cost. This could guide the search towards plans better than the terms representing the execution and temporal costs could be increased with respect to worth investigating is the use of dynamic coefficients to weight the terms of the action evaluation function. When we start a new search for a plan of better quality, the weights of LPG (Gerevini & Serina, 2002). This might be especially important for improving the types of graph modifications, some of which were implemented in the previous version of a "tabu list" similar to those presented in (Gerevini & Serina, 1999), as well as further extend the current version of the planner to handle plan metrics requiring to maximize the those already derived, which is purpose of the incremental process. Moreover, we intend to incremental plan-quality process. We have already mentioned a few directions that we are undertaking to improve our In addition, we intend to test other local search strategies based on the use of Another possible improvement of this process that is

1983) or STP-constraints (Dechter, Meiri, & Pearl, 1991). between similar actions to those that can be stated using Allen's Interval Algebra (Allen as well as the integration of temporal reasoning techniques to deal with temporal constraints temporal representation to handle upper and lower bounds on the possible action durations, Finally, a more general direction for future work concerns the treatment of a reacher

Acknowledgments

technique integrated into LPG is based on the same code developed for FF. some data structures of LPG are based on an extension of Jörg's code. The best-first search 3rd IPC, and to Jörg Hoffman who made the source code of FF available. The parser and Finally, thanks to Maria Fox who Derek Long for their hard work in the organization of the of the AI class at the University of Brescia who gave some contribution to this project. Lorini, and Stefano Orlandi for their support during the competition, and to all students Lazzaroni, Alessandro Saetti and Sergio Spinoni. Thanks also to Fabrizio Morbini, Valerio the competition in Toulouse, was very important. We would like to thank especially Marco dergraduate students. Their help in the implementation and test of LPG, before and during The development of LPG has been carried out with the valuable contributions of several un-

Appendix A: Mutex Relations in LPG

since it just applies the basic original definition of mutex relation (Blum & Furst, 1997). derives a set of mutex relations between facts, that are used by ComputeMutexActions to the set of its negative effects, and Pre(a) the set of its preconditions. ComputeMutexFacts gorithms given in Figure 13, where Add(a) denotes the set of the positive effects of a, Del(a)LPG precomputes a set of mutex relations for the input planning problem using the two alderive a set of relations between actions. The correctness of this second algorithm is obvious

have been considered (F^*) already been formulated (steps 12–18). The algorithm terminates when all possible facts relations (steps 7-11), and to possibly invalidate other potential mutex relations that have we consider every possible action a (step 5) to possibly generate a set of new potential mutex set F of all possible facts for the planning problem under consideration. At each iteration in which the facts of m are both true. between facts. A mutex relation m in M is global if there is no state that can be reached $(M^* = M)$. When the algorithm terminates M contains a set of global mutex relations from the initial state of the problem, using the operators of the domain under consideration, ComputeMutexFacts iteratively constructs a set M of potential mutex relations and the = F), and no new potential mutex relation can be generated

in (Gerevini & Schubert, 1998). (2) is based on the observation that, if f_1 is an effect of a, of a (steps 7, 10–11). (1) is a natural way of hypothesizing mutex relations that is used also from the application of a to a reachable state, f_2 and f_1 cannot be both true. $p \in Pre(a), f_2 \notin Add(a),$ and f_2 is mutually exclusive with p, then in any state resultant is a positive effect of a and the other is (potentially) mutually exclusive with a precondition them is a positive effect of a and the other is a negative effect (steps 7–9), or (2) one of them Given an action a, two facts f_1 and f_2 form a potential mutex relation m if (1) one of

mutually exclusive with no precondition of a. The first case if obvious, while the second an action containing the two facts of m among its positive effects (steps 13–14), or f_1 (f_2) produced by a (similarly if f_2 is a positive effect of a). is true in a state where a can be applied, then f_2 could persist from this state to the state can be explained as follows. If f_1 is a positive effect of a, and we cannot exclude that f_2 is an add-effect of an action a, f_2 (f_1) is not deleted by a, and f_2 (f_1) is (potentially) A potential mutex relation $m \in M$ between f_1 and f_2 becomes invalid if (1) there exists

by introducing an additional predicate not-p(x) if $\neg p(x)$ is needed and by formulating Add no explicit atomic negation is available in LPG's language, instead we model atomic negation and Delete effects correspondingly (this guarantees than not-p(x) and p(x) are mutex). Note that LPG handles negative preconditions as proposed in (Koehler et al., 1997). I.e.,

The next theorem states the correctness of our algorithms.

Theorem ComputeMutexFacts and ComputeMutexActions correctly compute a set of mutually exclusive relations between facts and actions respectively.

a state S from the initial state. and it can be proved by an inductive argument on the number k of actions applied to reach the two conditions under which a potential mutex relation is made invalid by the algorithm. the definition of mutually exclusive actions. Correctness of ComputeMutexFacts follows from **Proof.** Correctness of ComputeMutexActions is obvious, since it is a direct consequence of

$\mathsf{ComputeMutexFacts}(I,\mathcal{O})$

Output: A set of mutex relations between facts (M). Input: An initial state (I) and all ground operator instances (\mathcal{O}) ;

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21. return M.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 F^* \leftarrow I; F \leftarrow \emptyset; M \leftarrow \emptyset; M^* \leftarrow \emptyset; A \leftarrow \emptyset; while F^* \neq F \lor M^* \neq M
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 forall a \in \mathcal{O} such that Pre(a) \subseteq F^* and \neg(\exists p, q \in Pre(a) \land (p, q) \in M^*)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 F \leftarrow F^*; M \leftarrow M^*
                                                           \begin{array}{l} L \leftarrow Add(a) - New(a);\\ \textbf{forall } (i,\,q) \in M^* \text{ such that } i \in L\\ \textbf{if } q \notin Del(a) \land \neg (\exists\,p \in Pre(a) \land (p,\,q) \in M^*) \textbf{ then}\\ M^* \leftarrow M^* - \{(i,\,q),(q,\,i)\}; \qquad /^* \text{ Invalid mutex relation } */\\ \end{array}
                         A \leftarrow A \cup \{a\};
                                                                                                                                                                                                                                                                                  if a \notin A then
                                                       F^* \leftarrow F^* \cup New(a);
                                                                                                                                                                                                                                                                                                                                                                                                                                    forall f \in New(a)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                      New(a) \leftarrow Add(a) - F^*;
                                                                                                                                                                                                                                              for
all p, q \in Add(a) such that (p, q) \in M^*
                                                                                                                                                                                                                                                                                                          forall (p, q) \in M^* such that p \in Pre(a) and q \notin Del(a) M^* \leftarrow M^* \cup \{(f, q), (q, f)\}; /* Potential mutex relation */
                                                                                                                                                                                                                                                                                                                                                                                                         forall h \in Del(a)
                                                                                                                                                                                                             M^* \leftarrow M^* - \{(p, q), (q, p)\};
                                                                                                                                                                                                                                                                                                                                                                     M^* \leftarrow M^* \cup \{(f, h), (h, f)\};
                                                                                                                                                                                                                  /* Invalid mutex relation */
                                                                                                                                                                                                                                                                                                                                                                          /* Potential mutex relation */
```

ComputeMutexActions (M, \mathcal{O})

Input: A set of mutex relations between facts (M) and all ground operator instances (\mathcal{O}) ; Output: A set of mutex relations between actions (N).

```
forall (p, q) \in M
                                                            N \leftarrow \emptyset; \mathcal{O}^* \leftarrow \mathcal{O} extended with the no-op of every fact;
```

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9.
                           \dot{\infty}
                                                                         6.
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                                                                                                                         4
                                                                                                                                             ಭ
                                                                     for
all a \in \mathcal{O}^*
                                                                                                                                           forall a \in \mathcal{O}^* such that p \in Pre(a)
                                               forall p \in Pre(a)
                                                                                          for all b \in \mathcal{O}^* such that q \in Pre(b)
 N \leftarrow N \cup \{(a, b), (b, a)\}; /*
                       forall b \in \mathcal{O} such that p \in Del(b)
N \leftarrow N \cup \{(a, b), (b, a)\};
   /* Interference */
                                                                                            /* Competing needs */
```

10. 11.

forall $p \in Add(a)$

forall $b \in \mathcal{O}$ such that $p \in Del(b)$

 $N \leftarrow N \cup \{(a, b), (b, s)\};$

/* Inconsistent effects */

return N.

Figure 13: LPG's algorithm for computing the mutex relations.

relations involving two facts that are both true in the initial state. mutex relation for the initial state (S = I), because the algorithm cannot formulate mutex Induction base (k=0). It is easy to see that each element m in the output set M is a valid

mutex relation in any state reached by the application of n actions $(n \ge 1)$. Induction hypothesis (k = n). Suppose that any element m in the output set M is a valid

then remove it from M, contrary to our assumption that m belongs to the output M-set. the potential mutex relation between p and $f_2(f_1)$ to M, then it must be the case that it is then removed from M. It follows that, if some iteration adds (f_1, f_2) to M, steps 16–18 will steps 13–14 of ComputeMutexActions. Regarding case (ii), since we are assuming that S' is are both positive effects of a_{n+1} , or (ii) $f_1(f_2)$ is an add-effect of a_{n+1} , $f_2(f_1)$ is not deleted by a_{n+1} , and $f_2(f_1)$ is true in the state S' where a_{n+1} is applied. Case (i) is ruled out by action in this sequence. By the inductive assumption this can happen only if (i) f_1 and f_2 actions (because the two facts f_1 and f_2 of m are both true in S), and let a_{n+1} be the last that is not a valid mutex relation in a state S reachable by applying a sequence of n+1precondition p of a_{n+1} that is mutex with $f_2(f_1)$. Moreover, by the inductive assumption a reachable (consistent) state where $f_2(f_1)$ is true and a_{n+1} can be applied, it must exist no (p, f_2) $((p, f_1))$ cannot belong to the output M-set – if some iteration of the algorithm adds Induction step (k = n + 1). Assume that there exists an element m in the output set M

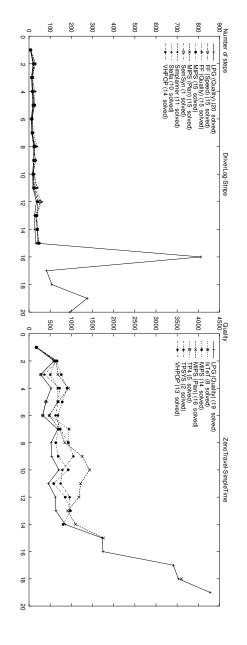
mum number of different facts, actions and potential mutex relations. \Box Termination of the two algorithms is guaranteed because there is always a finite maxi-

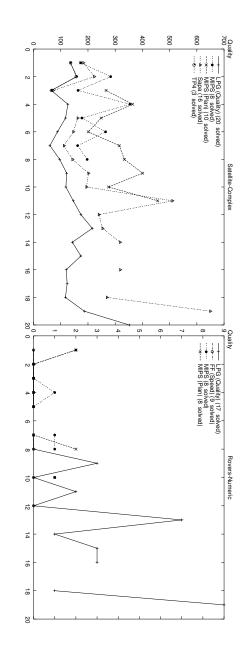
pairs" in M_0 , and there is no formal proof of correctness. mutex preconditions), while Bonet and Geffner derive M_0 using every operator instance. considering only applicable actions (i.e., actions with all preconditions in F^* and with nonin M_0 are different from the conditions used by ComputeMutexFacts to create M (especially verifies the set M through a forward process. The conditions under which a pair of facts is candidate mutex pairs, and then prune it, ComputeMutexFacts incrementally constructs and some significant differences. While Bonet and Geffner compute an initial large set M_0 of certain conditions on the preconditions and effects of the actions. However, there are also a set of pairs of mutex facts that are then possibly eliminated from the set according to between facts that has some similarities with ours. Both methods are based on hypothesizing Finally, their paper does not contain algorithmic details about the identification of "bad the condition in step 10). Bonet and Geffner (2001) proposed a method for deriving a set of mutex relations Moreover, our algorithm generates and tests the pairs of M

coded planners, like those of the domain Satellite Hand-Coded, the use of this option is memory, but each search step become slower. For this reason in the current version of precomputed). Preprocessing with lowmemory on becomes faster and requires much less actions at search time (while those between facts and between actions and no-ops are still can set an option of the planner (lowmemory) for computing the mutex relations between could be computationally very expensive. In order to cope with these cases, the user of LPG necessary. Currently we are studying an alternative method for computing mutex relations for the fully-automated planners, but for some of the problems designed for the handactions is prohibitive. This was never the case for the test problems of the 3rd IPC designed LPG this option is recommended only when the precomputation of mutex relations between For problems involving a very high number of actions, precomputing mutex relations

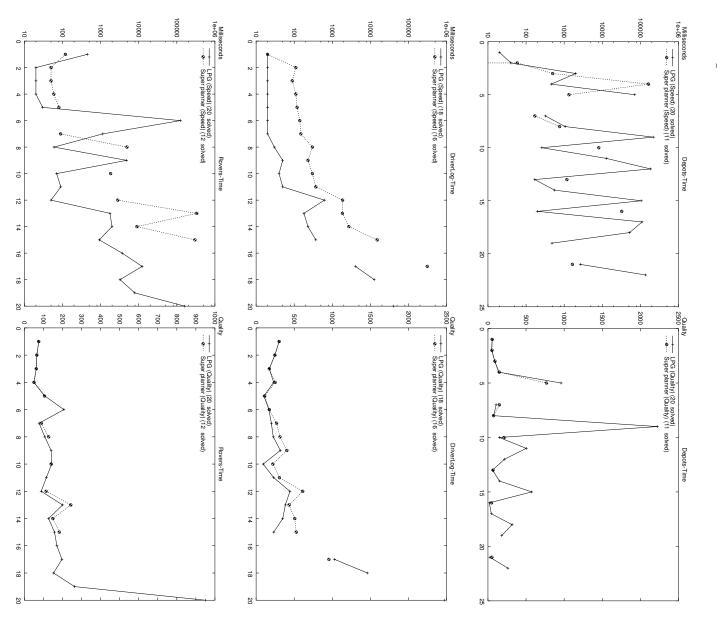
during search based on the use of state invariants computed by existing domain analysis tools, such as Discoplan (Gerevini & Schubert, 2000) or Tim (Fox & Long, 1998a). Finally, for domains involving numerical preconditions and effects, the set of mutex relations between actions computed by the algorithms of Figure 13 is extended using the definition of mutex relations for numeric domains given in (Fox & Long, 2001).

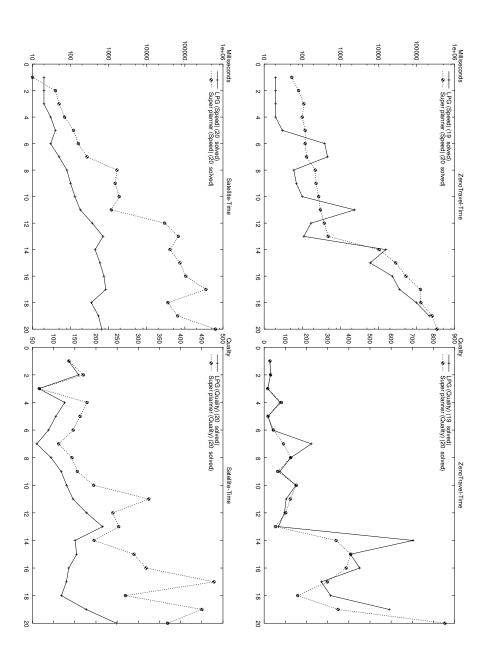
domains Appendix B: LPG-quality compared with all competitors in some





competition domains Appendix C: LPG and the SuperPlanner in the Time variant of the





Appendix D.1: Comparison of LPG-speed and the SuperPlanner

first was at least one order of magnitude lower than the second. system was considered much faster than the other one when the CPU-time required by the in which LPG-speed is much faster/slower than the SuperPlanner (5th/7th columns). A LPG-speed is faster/slower than the SuperPlanner (4th/6th columns); number of problems in terms of: number of problems solved (2nd and 3rd columns); number of problems in which ant of every domain tested using our local search techniques. The two systems are compared The following table shows the performance of LPG-speed and the SuperPlanner in every vari-

Appendix D.2: Comparison of LPG-quality and the SuperPlanner

the value of the metric expression, the better is the plan. plan quality is defined as the number of actions. In all problems considered, the lower is plan metric indicated in the problem specification, except for the Strips problem, where could not solve the corresponding problem). The quality of a plan is measured using the twice as good as the quality of π' , or if π exists and π' does not exist (because the system solution π' for the same problem derived by the other system if the quality of π is at least solution computed by the SuperPlanner (4th/6th columns); number of problems in which problems in which the quality of the solution computed by LPG is better/worse than the compared in terms of: number of problems solved (2nd and 3rd columns); number of variant of every domain tested using our local search techniques. The following table shows the performance of LPG-quality and the SuperPlanner in every (5th/7th columns). A solution π derived by a system is considered much better than the the solution of LPG-quality is much better/worse than the solution of the SuperPlanner The two systems are

| Total | DriverLog | Hard-numeric | Complex Satellite | Total | ZenoTravel | Satellite | Rovers | DriverLog | Depots | Numeric | Total | ZenoTravel | Satellite | Rovers | DriverLog | Depots | Time | Total | ZenoTravel | Satellite | Rovers | DriverLog | Depots | Simple-time | Total | ZenoTravel | Satellite | Rovers | DriverLog | Depots | ${f STRIPS}$ | | Domain | |
|-------|-----------|--------------|----------------------|-------|------------|------------------|----------|-----------|------------|---------|-------|------------|-----------|-----------|-----------|------------|------|-------|------------|-----------|-----------|-----------|------------|-------------|-------|------------|-----------|-----------|---------------|------------|--------------|-----------------------|-------------|-------------|
| 94.6% | 20 (100%) | | 20 (100%) | 86.3% | 20 (100%) | 12 (60%) | 17 (85%) | | | | 95.1% | 19 (95%) | _ | 20 (100%) | 18~(90%) | 20 (90.9%) | | 96% | 19~(95%) | | 20 (100%) | 18 (90%) | 21~(95.5%) | | 99% | 19~(95%) | 20 (100%) | 20 (100%) | 20 (100%) | 22 (100%) | | by LPG | solved | Problems |
| 80.3% | 16 (80%) | | 17 (85%) | 77.4% | 20 (100%) | 14 (70%) | 9~(45%) | 16 (80%) | 20 (90.9%) | | 77.4% | 20 (100%) | | 12~(60%) | 16~(80%) | 11 (50%) | | 70.6% | 16 (80%) | | 10 (50%) | 16~(80%) | 11~(50%) | | 95.1% | 20~(100%) | 20 (100%) | | $\overline{}$ | 22 (100%) | | the Super- Planner | solved by | Problems |
| 71% | 18 (90%) | | 19 (95%) | 52.9% | 9 (45%) | 12 (60%) | 8 (40%) | 15 (75%) | 10 (45.5%) | | 81.4% | 11 (55%) | _ | 18~(90%) | 17 (85%) | 17 (77.3%) | | 91.2% | 17 (85%) | | 20 (100%) | 17 (85%) | 19~(86.4%) | | 50% | 3~(15%) | 13~(65%) | 9~(45%) | | 12~(54.5%) | | the Super- Planner | better than | LPG |
| 21.9% | 5 (25%) | | 9 (45%) | 18.6% | 1 (5%) | 7 (35%) | 8 (40%) | | _ | | 25.5% | 0 (0%) | 5 (25%) | 8 (40%) | 4 (20%) | 9 (40.9%) | | 33.3% | 3 (15%) | | 10 (50%) | 3 (15%) | 11 (50%) | | 4.9% | 0 (0%) | 0 (0%) | | 5 (25%) | 0 (0%) | | the Super- Planner | better than | LPG much |
| 11.6% | 1 (5%) | | 1 (5%) | 17.6% | 6 (30%) | $\frac{4}{20\%}$ | 0 (0%) | 0 (0%) | | | 13.7% | | _ | 2(10%) | 1(5%) | 2(9.1%) | | 3.9% | 2(10%) | _ | 0 (0%) | 1(5%) | 1(4.5%) | | 13.7% | 9 (45%) | 0 (0%) | 1(5%) | 0 (0%) | 4 (18.2%) | | the Super- Planner | worse than | LPG quality |
| 2.7% | 0 (0%) | | 0 (0%) | 7.8% | 3 (15%) | | 0 (0%) | 0 (0%) | | | 2.9% | 3 (15%) | | | _ | 0 (0%) | | 0% | 0 (0%) | _ | 0 (0%) | 0 (0%) | 0 (0%) | | 0.9% | 1 (5%) | 0 (0%) | 0 (0%) | 0 (0%) | 0 (0%) | | the Super- Planner | worse than | LPG much |

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