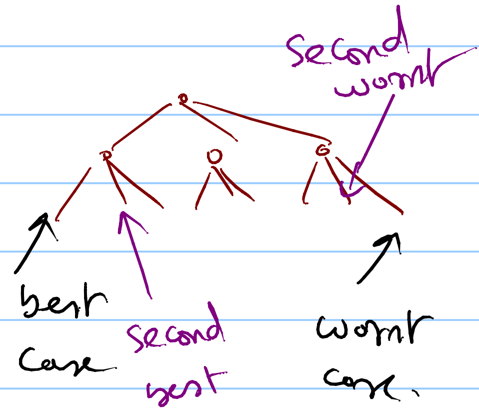
Note Title 9/14/2006

Showing that Ratio of nodes expanded by IDDFS to DFS is $\frac{b+1}{b-1}$ (when searching on a uniform tree with branching factor $\underline{b}$ where the solution occurs at last level $\underline{d}$)

① First let us compute the average number of nodes expanded by DFS

In the best case DFS expands

$\underline{d+1}$ nodes ——— ①




In the worst case DFS expands

$$1+b+\cdots+b^d = \frac{b^{d+1}-1}{b-1} \quad \text{——— ②}$$

In the average case it expands

$$\frac{① + ②}{2} \text{ nodes}$$

This makes sense only because the sum of second best and second worst will also be the same

$$= \left(\frac{b^{d+1}-1}{b-1} + d+1\right) \Big/ 2$$

$$= \boxed{\frac{b^{d+1}+bd+b-d-2}{2(b-1)}} \quad \text{——— ③}$$

avg # nodes expanded by DFS

Taken from Ginsberg's Essentials of AI?

Now we compute the avg num nodes expanded by DFS

we note that

Eqn 3 from before

① IDDFS expands the same number of nodes as DFS in the last iteration

② In the first $d-1$ iterations it expands the complete trees of that depth (since all goals are in the final level).

In the $J^{th}$ iteration, IDDFS expands $1 + b + \cdots + b^J = \frac{b^{J+1} - 1}{b-1}$ nodes

In the first $d-1$ iterations, it expands

$$\sum_{J=0}^{d-1} \frac{b^{J+1} - 1}{b-1} = \frac{1}{b-1}\left[ b\left( \sum_{J=0}^{d-1} b^J - \sum_{J=0}^{d-1} 1 \right)\right]$$

$$= \frac{1}{b-1}\left[ b\left(\frac{(b^d - 1)}{b-1}\right) - d \right]$$

$$= \frac{1}{b-1}\left[ b\left(\frac{b^d-1}{b-1}\right) - d \right]$$

$$= \frac{b^{d+1} - bd - b + d}{(b-1)^2} \quad \text{— (4)}$$

Adding (3) and (4) and simplifying we have total num expansions by IDDFS as

$$\frac{b^{d+2} + b^{d+1} + b^2 d + b^2 - 4bd - 5b + 3d + 2}{2(b-1)^2} \quad \text{— (5)}$$

for large $d$ this is dominated by

$$\frac{(b+1)\, b^{d+1}}{2(b-1)^2} \quad \text{— (5')}$$

For large $d$ the num nodes expanded by Dfs itself is dominated by

$$\frac{b^{d+1}}{2(b-1)} \quad — \quad 3'$$

So $$\frac{5}{3} \approx \frac{5'}{3'} \leftarrow \boxed{\frac{b+1}{b-1}}$$

Phew