PLANNING GRAPH BASED HEURISTICS FOR AUTOMATED PLANNING

by

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ABSTRACT

The main concern in automated planning is to construct a sequence of actions that achieves an objective given an initial situation of the world. Planning is hard; even the most restrictive case of automated planning, called classical planning, is PSPACE-complete in general. Factors affecting planning complexity are large search spaces, problem decomposition and complex action and goal interactions.

One of the most straightforward algorithms employed to solve classical planning problems is state-space search. In this algorithm, each state is represented through a node in a graph, and each arc in the graph corresponds to a state transition carried out by the execution of an action from the planning domain. A plan on this representation corresponds to a path in the graph that links the initial state of the problem to the goal state. The crux of controlling the search involves providing a heuristic function that can estimate the relative goodness of the states. However, extracting heuristic functions that are informative, as well as cost effective, remains a challenging problem. Things get complicated by the fact that subgoals comprising a state could have complex interactions. The specific contributions of this work are:

An underlying framework based on planning graphs that provides a rich source for extracting distance-based heuristics for disjunctive planning and regression state-space planning.

Extensions to the heuristic framework to support the generation of parallel plans in state-space search. The approach introduced generates parallel plans online using planning graph heuristics, and plan compression techniques; and

The application of state-space planning to cost-based over-subscription planning problems. This work extends planning graph heuristics to take into account real execution

costs of actions and goal utilities, using mutex analysis to solve problems where goals have complex interactions.

Beyond the context of planner efficiency and impressive results, this research can be best viewed as an important step towards the generation of heuristic metrics that are informative as well as cost effective not only for state-space search but also for any other planning framework. This work demonstrates that state-space planning is a viable alternative for solving planning problems that originally were excluded for taking into consideration given their combinatorial complexity.

To my family:

My dear wife Claudia, and little Romeo who are my inspiration.

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CHAPTER 1

Introduction

Planning in the general case can be seen as the problem of finding a sequence of actions that achieves a given goal from an initial situation of the world [Russell and Norvig, 2003]. Planning in fully observable, deterministic, finite, static and discrete domains is called *Classical Planning* [Ghallab *et al.*, 2004; Russell and Norvig, 2003; Kambhampati *et al.*, 1997], and although, real world problems may be far more complex than those represented by classical planning, it has been shown that even this restrictive class of propositional planning problems is PSPACE-complete in general [Bylander, 1994]. Therefore, one of the main challenges in planning is the generation of heuristic metrics that can help planning systems to scale up to more complex planning problems and domains. Such heuristic metrics have to be domain-independent in the absence of control knowledge in order to work across different plan synthesis algorithms and planning domains, which increases the complexity of finding efficient and flexible estimates.

More formally speaking, a planning problem can be seen as a three-tuple $\mathcal{P} = (\Omega, G, I)$, where Ω represents the set of deterministic actions instantiated from the problem description, G is a goal state, and I is the initial state of the problem. A plan ρ can be seen as a sequence of actions $a_1, a_2, ..., a_n$ which, when applied to the initial state I of the

problem, achieves the goal state G [Ghallab *et al.*, 2004; Russell and Norvig, 2003]. Each action $a_i \in \Omega$ has a set of conditions that must be true for the action to be applicable, such conditions are described in terms of a precondition list $Prec(a_i)$. The effects of the actions $Eff(a_i)$ are described in two separate lists, an add list $Add(a_i)$ that specifies the conditions that the action makes true, and a delete list $Del(a_i)$, which describes those conditions that the action negates from the current state of the world.

One of the most efficient planning frameworks for solving large deterministic planning problems is state-space planning [Bonet *et al.*, 1997; Bonet and Geffner, 1999; Hoffmann and Nebel, 2001; Do and Kambhampati, 2001; Nguyen *et al.*, 2002; Gerevini and Serina, 2002], which explicitly searches in the space of world states using heuristics to evaluate the goodness of them. The heuristic can be seen as estimating the number of actions required to reach a state, either from the goal G or the initial state I. The main challenge of course is to design such heuristic function h that will rank the states during search. Heuristic functions should be as informative as possible, as well as cheap to compute. However, finding the correct trade-off could be as hard as solving the original problem [Ghallab *et al.*, 2004]. Things get complicated by the fact that subgoals comprising a goal state could have complex interactions. There are two kinds of interactions among subgoals, negative and positive [Nguyen *et al.*, 2002]. Negative interactions happen when the achievement of a subgoal precludes the achievement of another subgoal. Ignoring this type of interaction would normally underestimate the cost of achievement. Positive interactions occur when the achievement of a subgoal reduces the cost of achieving another one. Ignoring positive interactions would overestimate the cost returned by the heuristic, making it inadmissible. In consequence, heuristics that make

strong assumptions (relaxations) about the independence of subgoals, often perform badly in complex problems. In fact, taking into account such interactions to compute admissible heuristics in state-space planning remains a challenging problem [Bonet and Geffner, 1999; Nguyen *et al.*, 2002].

This dissertation presents our work on heuristic planning. More specifically, our research demonstrates the scalability of state-space planning techniques in problems where their combinatorial complexity previously excluded state-space search for taking it into consideration (e.g., parallel planning, over-subscription planning). The main contribution of our work is the introduction of a flexible and effective heuristic framework that carefully takes into account complex subgoals interactions, producing more informative heuristic estimates. Our approach, based on planning graphs [Blum and Furst, 1997], computes approximate reachability estimates to guide the search during planning. Furthermore, as we will discuss later, our heuristic framework is flexible enough to be applied to any plan synthesis algorithm.

This work will show first that the planning graph data structure of *Graphplan* is an effective medium to automatically extract reachability information for any planning problem. It will show then how to use such reachability information to develop distance-based heuristics directly in the context of *Graphplan*. After that, this research will show that planning graphs are also a rich source for deriving effective and efficient heuristics, more sensitive to subgoals interactions, for controlling state-space search. In addition to this, methods based on planning graphs to control the cost of computing the heuristics and limit the branching factor of the search are also introduced. Extensions to our heuristic framework are made to support the generation of parallel plans in state-space search [Sanchez and

Kambhampati, 2003a]. Our approach generates parallel plans online using distance-based heuristics, and improves even further the quality of the solutions returned by using a plan compression algorithm.

Finally, we will also show the applicability of our heuristic framework to cost-based sensitive problems. More specifically, we will address the application of heuristic state-space planning to partial satisfaction (over-subscription) planning problems. Over-subscribed problems are those in which there are many more objectives than the agent can satisfy given its resource limitations, constraints or goal interactions. Our approach introduces a greedy algorithm to solve cost-sensitive partial satisfaction planning problems in the context of state-space search, using mutex analysis to solve over-subscribed problems where goals have complex interactions. We will present extensive empirical evaluation of the application of our planning graph based techniques across different domains and problems.

Beyond the context of planner efficiency, and impressive results, our current work can be best viewed as an important step towards the generation of heuristic metrics that are informative as well as cost effective not only for state-space search but also for any other planning framework. This work demonstrates then that planning graph based heuristics are highly flexible, and successful for scaling up plan synthesis algorithms.

The remainder of this chapter highlights our specific research contributions and the overall organization of this dissertation.

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1.1. Specific Research Contributions

The contributions of this dissertation can be divided in two major directions. In the first one, we demonstrate that the planning graph data structure from *Graphplan* is a rich source for extracting very effective heuristics, as well as important related information to control the cost of computing such heuristics and limit the branching factor of the search. Specifically, we will show the effectiveness of such heuristics in the context of regression state-space search by introducing two efficient planners that use them, *AltAlt* and *AltAlt*^{p}.[1] Part of *AltAlt*'s work has been presented at KBCS-2000 [Sanchez *et al.*, 2000], and has been also published by the *Journal of Artificial Intelligence* [Nguyen *et al.*, 2002]. *AltAlt*^{p}'s work has been presented at IJCAI-2003 [Sanchez and Kambhampati, 2003b], and it has been published by the *Journal of Artificial Intelligence Research* [Sanchez and Kambhampati, 2003a]. The reachability information from the planning graph has also been applied to the backward search of *Graphplan* itself. This work has been presented at AIPS-2000 [Kambhampati and Sanchez, 2000].

In the second direction, we will show that state-space planning can be successfully applied to more complex planning scenarios by adapting our heuristic framework. We will introduce a greedy state-space search algorithm to solve *Partial Satisfaction Cost-sensitive* (Over-subscription) problems. This time, our heuristic framework is extended to cope with cost sensitive information. This work has developed two planning systems *AltAlt*^{ps} and *AltWlt* that solve over-subscription planning with respect to the PSP Net Benefit

[1] Preliminary work on *AltAlt* was presented by Xuanlong Nguyen at AAAI-2000 [Nguyen and Kambhampati, 2000].

problem. [2] The work on *AltAlt*^{ps} has been presented in WIPIS-2004 [van den Briel *et al.*, 2004b] and in AAAI-2004 [van den Briel *et al.*, 2004a]. Extensions to *AltAlt*^{ps} to handle complex goal interactions and multiple goal selection was presented at ICAPS-2005 [Sanchez and Kambhampati, 2005].

1.2. Thesis Organization

The next Chapter presents a brief background on automated planning and its representation. We provide a description of classical planning, the specific planning substrate that this dissertation mostly deals with. We also introduce state-space plan synthesis algorithms, highlighting the need for heuristic support in planning.

In Chapter 3, we introduce the notion of distance-based heuristics in *Graphplan*. We show how these estimations can naturally be extracted from planning graphs, and use them to guide *Graphplan*'s own backward search. Then, we explain how we can further extract more aggressive planning graph heuristics and apply them to drive regression state-space search. We also show that planning graphs themselves are an effective medium for controlling the cost of computing the heuristics and reducing the branching factor of the search.

Next Chapter, we demonstrate the applicability of state-space search to parallel planning by extending our heuristic framework. Our approach is sophisticated in the sense that parallelizes partial plans online using planning graph estimations. Our empirical eval-

[2] Curious readers may advance to Chapter 5 for a description of PSP Net Benefit.

uation shows that our approach is an attractive tradeoff between quality and efficiency in the generation of parallel plans.

Finally, Chapter 5 exposes state-space planning to *Partial Satisfaction* problems [Haddawy and Hanks, 1993], where the planning graph heuristics are adjusted to take into account real execution costs of actions and goal utilities. This chapter also presents techniques to account for complex goal interactions using mutex analysis from the planning graph. Chapter 6 discusses related work, and Chapter 7 summarizes the contributions of this dissertation and future directions.

CHAPTER 2

Background on Planning and State-space Search

Automated planning can be seen as the process of synthesizing goal-directed behavior. In other words, planning is the problem of finding a course of actions that deliberatively transforms the environment of an intelligent agent in order to achieve some predefined objectives. Automated planning not only involves action selection, but also action sequencing, entailing during this process rational behavior. Therefore, one of the main motivations behind automated planning is the design and development of autonomous intelligent agents that can interact with humans [Ghallab *et al.*, 2004].

There are many forms of planning given that there are many different problems in which planning could be applied. In consequence, planning problems could be addressed using domain-specific approaches, in which each problem gets solved using a specific set of techniques and control knowledge related to it. However, domain-specific planning techniques are hard to evaluate and develop given that they are specific to a unique agent structure, and in consequence, their applicability is very limited. For all these reasons, unless stated otherwise, this dissertation is concerned in developing domain-independent planning tools that can be applicable to a more general range of planning problems. Domain-independent planners take as input an abstract general model of actions, and a problem definition, pro-

ducing a solution plan. Depending of the problem, a solution plan could be sets of actions sequences, policies, action trees, task networks, variable assignments, etc.

Planning is hard, some of the main factors that increase planning complexity are large search spaces, lack of heuristic guidance, problem decomposition and complex goal and action interactions. However, plan synthesis algorithms have advanced enough to be useful in a variety of applications. Including among these NASA space applications [RAX, 2000; Jonsson *et al.*, 2000; Ai-Chang *et al.*, 2004], aviation (e.g, flight planning software), DoD applications (e.g, mission planning), planning with workflows [Srivastava and Koehler, 2004], planning and scheduling integration [Kramer and Giuliano, 1997; Frank *et al.*, 2001; Smith *et al.*, 2000; 1996; Chien *et al.*, 2000], grid computing [Blythe *et al.*, 2003], autonomic computing [Ranganathan and Campbell, 2004; Srivastava and Kambhampati, 2005], logistics applications and supply chain management (e.g. transportation, deployment, etc), data analysis, process planning, factory automation, etc.

The success of plan synthesis algorithms in the last few years is mainly due to the development of efficient and effective heuristics extracted automatically from the problem representation, which help planning systems to improve their search control. The primary goal of this dissertation is to show the impact of our work on this planning revolution by demonstrating empirically and theoretically that state-space planning algorithms can scale up to complex problems, when augmented with efficient and effective heuristics. Planning graphs provide rich reachability information that can be used to derive estimates that can be used across different planning problems.

The rest of this Chapter is organized as follows, in the next Section a brief background on the many complexities of planning is provided, putting special emphasis on the

substrate of planning that this research mostly deals with (i.e., classical planning) and its representation. After that, state-space plan synthesis algorithms are presented, highlighting their need for heuristic support in order to scale up to complex planning problems.

2.1. The Classical Planning Problem

The planning problem involves manipulation of the agent's environment in an intelligent way in order to achieve a desired outcome. Under this scenario, the complexity of plan synthesis is directly linked to the capabilities of the agent and the restrictions on the environment. This dissertation considers only environments that are fully observable, static, propositional, finite and in which the agent's execution of actions are instantaneous (discrete) and deterministic. Plan synthesis under these conditions is known as the classical planning problem [Russell and Norvig, 2003; Ghallab *et al.*, 2004; Kambhampati, 1997], see Figure 1 reproduced from [Kambhampati, 2004]:

- Fully observable: the environment is fully observable if the agent has complete and perfect knowledge to identify in which state of the world it is.
- Static: The environment is static if only responds to the agent's changes.
- Propositional: Planning states are represented with boolean state variables.
- Finite: The whole planning problem can be represented with a finite number of states.
- Deterministic: Each possible action of the agent, when applicable to a single state, leads to a well defined other single state.

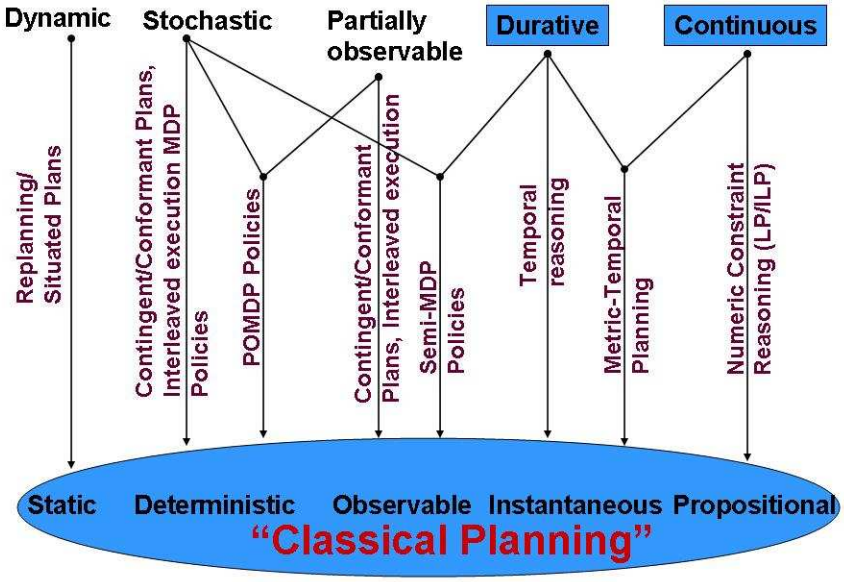


Figure 1. Planning substrates

- Instantaneous (Discrete): Agent's actions do not have durations. They are instantaneous state transitions.

Although, classical planning appears to be restrictive for more real world problems, it is still computationally very hard, PSPACE-complete or worse [Erol *et al.*, 1995; Bylander, 1994; Ghallab *et al.*, 2004]. We can see in Figure 1 some of these planning environments. Notice that a particular extension over instantaneous actions is when actions have durations, but still the planning problem could remain classical (if the environment is static, deterministic and fully observable). Some adaptations of the heuristics estimates discussed in this research have been implemented to cope with these types of problems [Do and Kambhampati, 2003; Bryce and Kambhampati, 2004].

2.2. Plan Representation

The classical planning problem involves selection of actions as well as sequencing decisions to change the environment of the agent. Therefore, one way of representing the classical planning problem is to use general models that reflect the nature of dynamic systems. One such a model is a *state-transition system* [Dean and Wellman, 1991; Ghallab *et al.*, 2004]. A state-transition planning system can be seen as a 3-tuple $\Upsilon = (S, A, \gamma)$, where:

- S is a finite set of states;
- A is a finite set of deterministic actions; and
- γ is ternary relation in terms of S x A x S, which represents the state-transition function showing that there is a transition from state s_i to state s_j with action a_k.

A state-transition system Υ can be seen as a directed graph, where the states S correspond to nodes in the graph, and actions in A correspond to the arcs in the graph, labeling the transitions between the nodes. Under this representation, finding a plan in deterministic environments is equivalent to finding paths in the graph corresponding to the transition system. Transition system models are commonly called "explicit" models because they explicit enumerate the set of all possible states and transitions. Unfortunately, such description is impossible in large and complex planning problems. In general, factored models are needed, in which the planning problem is more compactly represented, and in which states and their transitions are computed on-the-fly. One of such representations

is based on State-variable models. Next subsection introduces one model based on binary state-variables, which constitutes the most known representation for classical planning.

2.2.1. Binary State-variable Model. One of the most known representations for classical planning is the *Binary State-variable* model. On this model, states of the world are represented by binary state-variables. In other words, each state is a conjunction of logical atoms (propositional literals) that can take true or false values. Under this representation, actions are modeled as planning operators that change the truth values of the state literals, they are in fact considered as state transformation functions. For the purposes of this work, literals are completely ground and function free.

Most work in classical planning has followed the state-variable model using the STRIPS representation [Fikes and Nilsson, 1971; Lifschitz, 1986]. In STRIPS, a planning state is conformed of a conjunction of positive literals. For simplicity, we consider the *closed-world* assumption [Russell and Norvig, 2003], meaning that any literals that are not present in a particular state have false values. A planning problem using STRIPS is then specified by:

- A complete initial state,
- A partially specified goal state, in which non-goal literals are not specified; and
- A set of ground actions. Each action is represented in terms of its preconditions, which consist of a set of conditions (literals) that need to be true in the state for the action to be executed; and a set of effects (positive as well as negative) that describes how the state changes when the action gets executed.

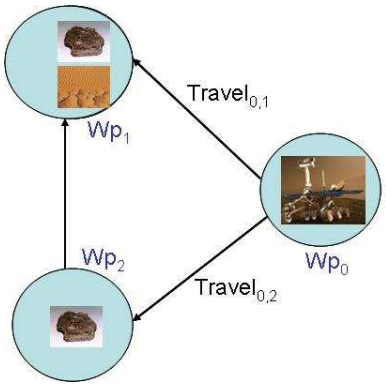


Figure 2. Rover planning problem

The most accepted standard language to represent planning problems inspired by the STRIPS representation is PDDL [Committee, 1998]. [1] In PDDL, planning problems are usually described using two components. The first component describes the literals that conform the planning domain, their types, any relations among them, and their values. It also describes the literals that conform the initial state of the problem as well as the top level goal state. The second component is an operator file that describes the skeleton for the actions in terms of their parameters, preconditions, and effects. We can see in Figure 2 a problem from the Rover domain [Long and Fox, 2003], and in Example 2.2.1 a description of it using the PDDL language.

Example 2.2.1

Suppose that we want to formulate a rover planning problem in which there are three locations or waypoints (`wp0, wp1, wp2`), one rover (`rover0`), one store (`store0`), and one lander (`general`). There are two types of samples (i.e., rock and soil). The problem is to travel across different waypoints to collect the samples and send the data back to the

[1] The Planning Domain Definition Language

lander. We can see in Figure 2 that there are only three samples to collect, two in waypoint one, and one in waypoint two. For this problem, we have the following PDDL description:

Rover problem.pddl

```
(define (problem roverExample) (:domain Rover)
(:objects general rover0 store0
wp0 wp1 wp2)
(:init
    (visible wp0 wp1)
    (visible wp1 wp0)
    (visible wp0 wp2)
    (visible wp2 wp0)
    (visible wp1 wp2)
    (visible wp2 wp1)
    (rover rover0) (store store0) (lander general)
    (waypoint wp0) (waypoint wp1) (waypoint wp2)
    (atsoilsample wp1)
    (atrocksample wp1) (atrocksample wp2)
    (channelfree general) (at general wp0)
    (at rover0 wp0) (available rover0)
    (storeof store0 rover0) (empty store0)
    (equippedforsoilanalysis rover0) (equippedforrockanalysis rover0)
    (cantraverse rover0 wp0 wp1)
    (cantraverse rover0 wp0 wp2)
    (cantraverse rover0 wp2 wp1))
(:goal (and
    (communicatedsoildata wp1)
    (communicatedrockdata wp1)
    (communicatedrockdata wp2))))
```

end *Problem definition*;

The second component is the domain file, which describes the operators that are applicable in the planning problem. This file describes basically the dynamics of the planning domain by specifying the literals that each operator requires, and also those that they affect. Here is a partial example on the rover domain:

```
Rover Domain.pddl
    (:requirements :strips)
    (:predicates (at ?x ?y) ...)
    (:action navigate
      :parameters ( ?x ?y ?z)
      :precondition
           (and (rover ?x) (waypoint ?y) (waypoint ?z)(at ?x ?y)
           (cantraverse ?x ?y ?z) (available ?x)(visible ?y ?z))
      :effect
           (and (not (at ?x ?y)) (at ?x ?z)))
    (:action samplesoil
      :parameters ( ?x ?s ?p)
      :precondition
           (and (rover ?x) (store ?s) (waypoint ?p)
           (at ?x ?p) (atsoilsample ?p) (empty ?s)
           (equippedforsoilanalysis ?x) (storeof ?s ?x))
      :effect
           (and (not (empty ?s)) (not (atsoilsample ?p))
           (full ?s) (havesoilanalysis ?x ?p)))
    (:action communicatesoildata
      :parameters (?r ?l ?p ?x ?y)
      :precondition
           (and (rover ?x) (lander ?l) (waypoint ?p) (waypoint ?x)
           (waypoint ?y) (at ?r ?x) (at ?l ?y) (havesoilanalysis ?r ?p)
           (visible ?x ?y) (available ?r) (channelfree ?l))
      :effect
           (communicatedsoildata ?p))
    (:action drop
      :parameters ( ?x ?y)
      :precondition
           (and (rover ?x) (store ?y) (storeof ?y ?x) (full ?y))
      :effect
           (and (not (full ?y)) (empty ?y)))
end Domain definition;
```

Once we have the domain and problem description in PDDL, they are used to compute the set of ground actions that the planner manipulates in order to find a solution to the problem. This step during the planning process is commonly called *plan synthesis*, and there are a variety of planning algorithms that perform it. In the next Section, we briefly

discusses some of the most popular algorithms, putting special emphasis on *state-space search* algorithms, in which our planning solutions are based.

2.3. Heuristic State-space Plan Synthesis Algorithms

Algorithms that search on the space of world states are maybe the most straightforward algorithms used to solve classical planning problems. In these algorithms, each state of the world is represented through a node in a graph structure, and each arc in the graph corresponds to a state transition carried out by the execution of a single action from the planning domain. In consequence, in *state-space* planning each state is represented as a set of propositions (or subgoals). A plan on this representation would correspond to a path in the graph that links the initial state of the problem to the goal state. We can see in Figure 3 a subset of the search space unfolded from the initial state specified in Figure 2, and described by Example 2.2.1. Notice that the initial and goal states are pointed out in the figure, and specified by S_0 = {`at(rover0,wp0), atrocksample(wp1), atsoilsample(wp1), atrocksample(wp2), at(general,wp0),...`}, and G = {`communicatedsoildata(wp1), communicatedrockdata(wp1), communicatedrockdata(wp2)`}.

As mentioned before, a planning problem in state-space gets also represented as a three-tuple $\mathcal{P} = (\Omega, G, S_0)$. We are given a complete initial state S_0, a goal state G that could be partially specified, and a set of deterministic actions Ω which are modeled as state transformation functions. As mentioned earlier, each action $a \in \Omega$ has a precondition list, add list and delete list (effects), denoted by $Prec(a), Add(a)$, and $Del(a)$, respectively. The planning problem is concerned with finding a plan ρ, e.g a totally ordered sequence

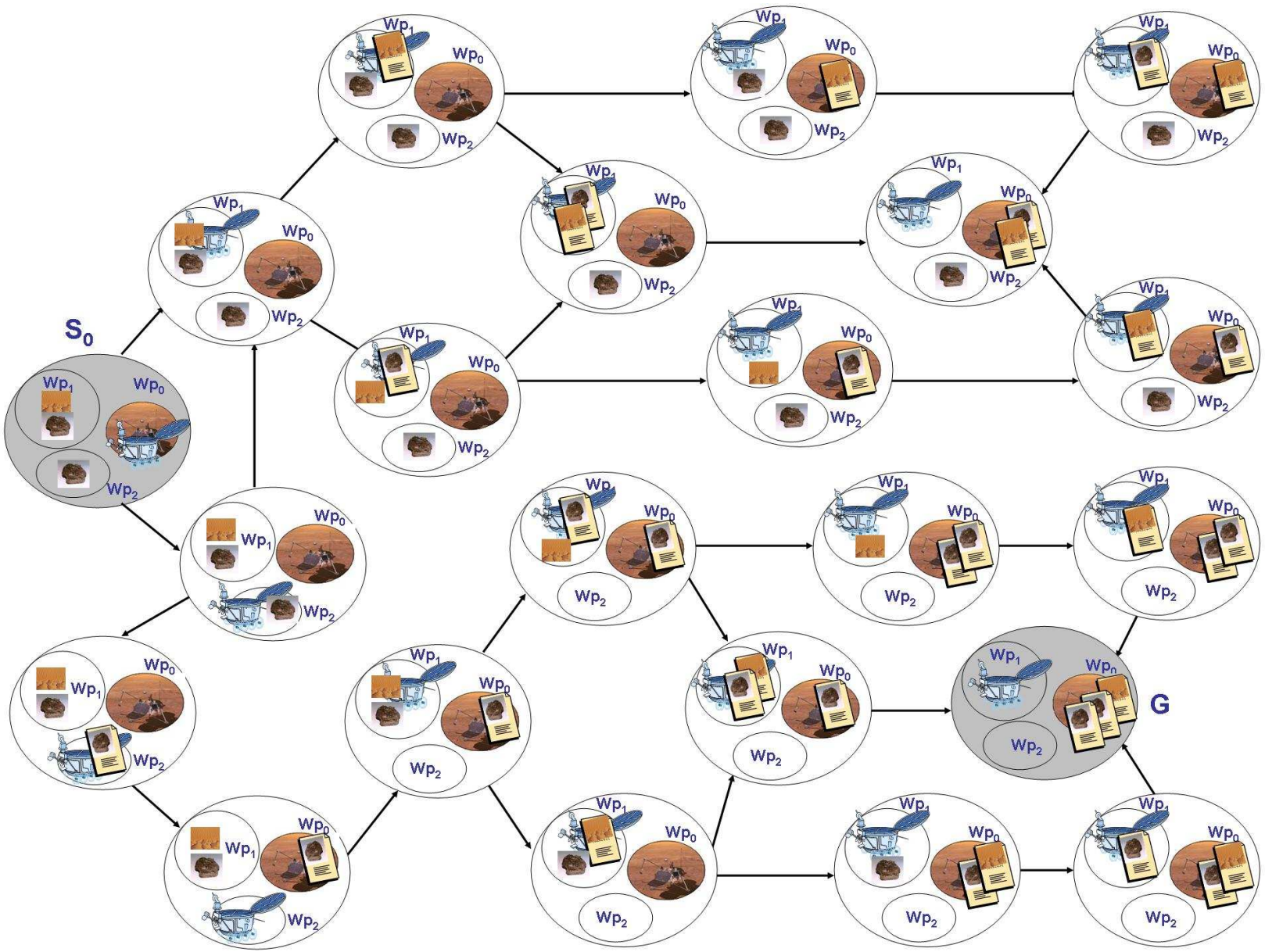


Figure 3. Partial rover state-space

Algorithm *ForwardSearch*(S_0, G, Ω)
  $S \leftarrow S_0$
  $\rho \leftarrow \emptyset$
  loop
    if$(G \subseteq S)$ **return** ρ
    $Q \leftarrow \{a | a \in \Omega,$ *and applicable in* $S\}$
    if$(Q = \emptyset)$ **return** Failure
    nondeterministally choose $a \in Q$
    $S \leftarrow Progress(S, a)$
    $\rho \leftarrow \rho.a$
End *ForwardSearch*;

Figure 4. Forward-search algorithm

of actions in Ω,[2] that when applied to the initial state S_0 (and executed) will achieve the goal G. Given the representation of the planning problem, there are two obvious ways of implementing *state-space* planning. *Forward-search* and *Backward-search*.

2.3.1. Forward-search. Starts from the initial state S_0, trying to find a state S' that satisfies the top level goals G. In Forward search, we progress the state space through the application of actions. An action a is said to be applicable to state S if $Prec(a) \subseteq S$. The result of progressing an action a over S is defined using the following progression function:

$$Progress(S, a) := (S \cup Add(a)) \setminus Del(a) \tag{2.1}$$

The states produced by the progression function 2.1 are consistent and complete given that the initial state S_0 is completely specified. However, heuristics have to be re-computed at every new state during search, which could be very expensive. We can see in Figure 4 a description of the Forward-search algorithm. It takes as input a planning prob-

[2]We will relax this restriction later in Chapter 4 when we consider parallel state-space planning.

lem $P = (S_0, G, \Omega)$ specified in terms of the initial and goal states, and the set of actions in the domain [Ghallab *et al.*, 2004]. The algorithm returns a plan ρ if there is a solution, or failure otherwise. The nondeterministic choice of the next state to progress in the algorithm is usually manipulated heuristically. Otherwise, it would be impossible to search the large *state-space* of complex planning problems. In progression, the heuristic function h over a state S is the cost estimate of a plan that achieves G from that state. We could check correctness of a plan ρ by progressing the initial state S_0 through the sequence of actions $a \in \rho$, checking that G is present in the final state of the sequence. The Forward-search classical planning algorithm is sound and complete [Ghallab *et al.*, 2004]

Example 2.3.1

As an example of how the Forward-search algorithms works, consider the initial state S_0 shown in Figure 2, and the domain description introduced in our last example. It can be seen that the partial action sequence ρ = { `navigate(rover0,wp0,wp2)`, `samplerock(rover0,store0,wp2)`, `communicaterockdata(rover0,general,wp2,wp2,wp0)` }, produces the resulting state S' = { `at(rover0,wp2)`, `full(store0)`, `haverockanalysis(rover0,wp2)`, `communicatedrockdata(wp2)` } if executed from S_0, constituting the path represented in the partial graph shown in Figure 5.

2.3.2. Backward-search. Although the Forward-search algorithm generates only consistent states, the branching factor of its search can be quite large. The main reason for this is that at each iteration the algorithm progresses all the actions in the domain that are

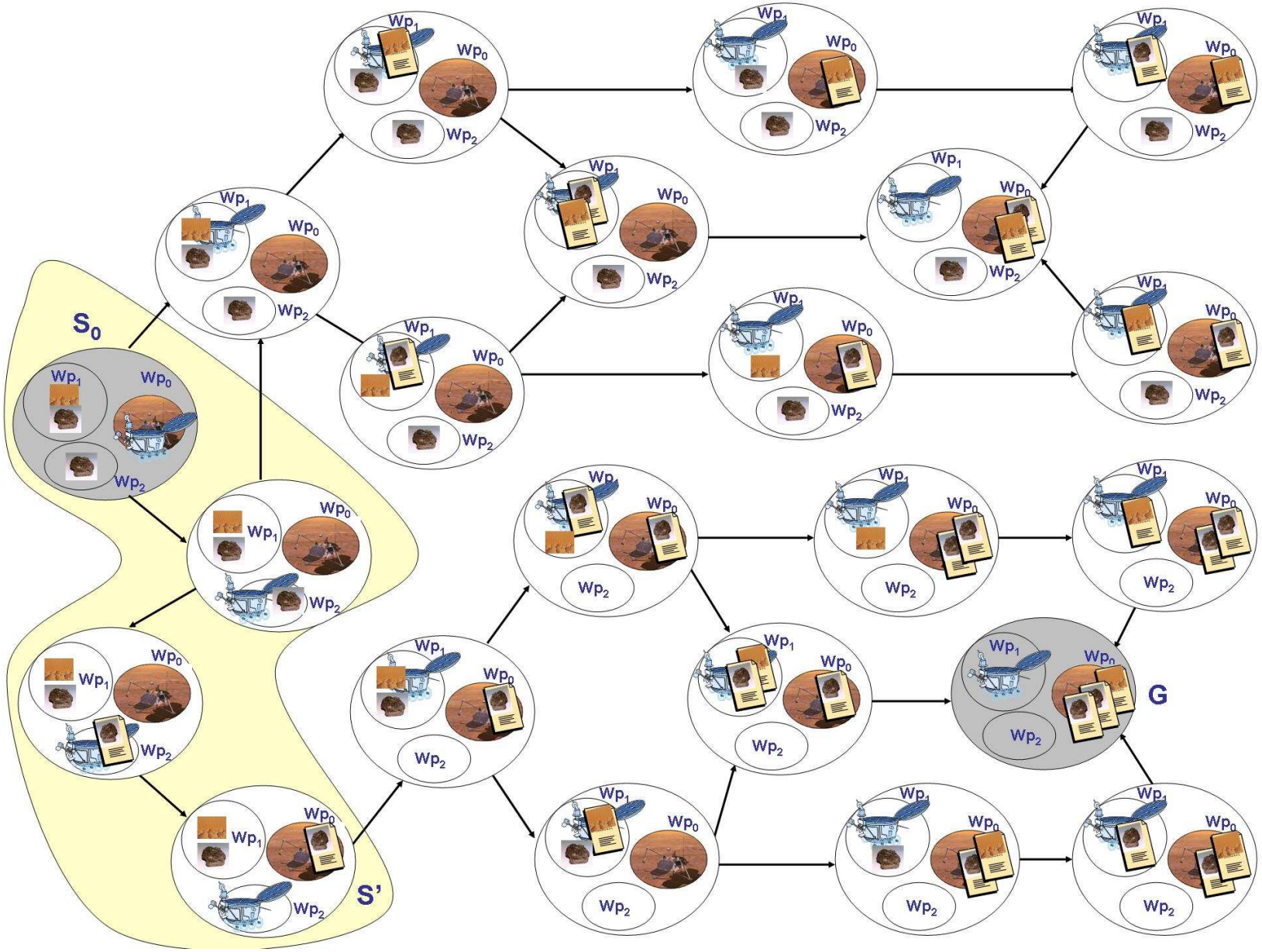


Figure 5. Execution of partial plan found by Forward-search

applicable to the current state. The problem is that many of these actions may not be even relevant for achieving our goals, this is called the irrelevant action problem [Russell and Norvig, 2003]. On the other hand, the Backward-search algorithm considers only relevant actions to the current subgoal, making the search more goal oriented. An action is said to be relevant to a current state, if it achieves at least one of the literals (subgoals) in it. The idea with Backward-search is to start from the top-level goal definition, and apply inverses of actions to produce pre-conditions. The algorithm stops when our current state is subsumed by the initial state. More formally speaking, in backward state-space search, an action a is said to be regressable to state S if:

- Action is relevant, $Add(a) \cap S \neq \emptyset$, and;
- it is consistent, $Del(a) \cap S = \emptyset$.

Then, the regression of S over an applicable action a is defined as:

$$Regress(S, a) := (S \setminus Add(a)) \cup Prec(a) \tag{2.2}$$

The result of regressing a state S over an action a represents basically the set of goals that still need to be achieved before the application of a, such that everything in S would have been achieved once a is applied. We can see the overall description of the Backward state-space search algorithm in Figure 6. The Backward-search algorithm is also sound and complete [Pednault, 1987; Weld, 1994].

Even though the branching factor of Backward-search gets reduced to the application of relevant actions, it can still be large. Moreover, Backward-search works on states that

```
Algorithm BackwardSearch(S_0, G, Ω)
    S ← G
    ρ ← ∅
    loop
        if(S ⊆ S_0) return ρ
        Q ← {a|a ∈ Ω, and regressable in S}
        if(Q = ∅) return Failure
        nondeterministally choose a ∈ Q
        S ← Regress(S, a)
        ρ ← a . ρ
End BackwardSearch;
```

Figure 6. Backward-search algorithm

are partially specified, producing more spurious states. Therefore, heuristic estimates are also needed to speed up search. In regression, heuristic functions are computed only once from the single initial state, representing the cost estimate of a plan that achieves a fringe state S from the initial state S_0.

2.4. Heuristic Support for State-space Planning

The efficiency and quality of state-space planners depend critically on the informedness and admissibility of their heuristic estimators. The difficulty of achieving the desired level of informedness and admissibility of the heuristic estimates is due to the fact that subgoals interact in complex ways. As mentioned before, there are two kinds of subgoal interactions: negative interactions and positive interactions. Negative interactions happen when achieving one subgoal interferes with the achievement of some other subgoal. Ignoring this kind of interactions would normally underestimate the cost, making the heuristic uninformed. Positive interactions happen when achieving one subgoal also makes it easier to achieve other subgoals. Ignoring this kind of interactions would normally overestimate

the cost, making the heuristic estimate inadmissible. For the rest of this section we will demonstrate the importance of accounting for subgoal interactions in order to compute more informed heuristic functions. We do so by examining the weakness of heuristics such as those used by HSP-r [Bonet *et al.*, 1997], which ignore these subgoal interactions.

The HSP-r planner is a regression state-space algorithm. The heuristic value h of a state S is the estimated cost (number of actions) needed to achieve S from the initial state S_0. In HSP-r, the heuristic function h is computed under the assumption that the propositions constituting a state are strictly independent. Thus the cost of a state is estimated as the sum of the cost for each individual proposition making up that state.

$$h(S) \leftarrow \sum_{p \in S} h(p) \tag{2.3}$$

Where the heuristic cost $h(p)$ of an individual proposition p is computed using an iterative procedure that is run to fixpoint as follows. Initially, p is assigned a cost of 0 if it is in the initial state S_0, and ∞ otherwise. For each action $a \in \Omega$ that adds p, $h(p)$ is updated as:

$$h(p) \leftarrow min\{h(p), 1 + h(Prec(a))\} \tag{2.4}$$

The updates continue until the h values of all the individual propositions stabilize. Because of the independence assumption, the sum heuristic turns out to be inadmissible (overestimating) when there are positive interactions between subgoals. Sum heuristic is also less informed (significantly underestimating) when there are negative interactions between subgoals.

We can follow our working example 2.3.1 to see how these limitations affect the *sum* heuristic. Suppose that we want to estimate the cost of achieving the state $S = \{$`communicatedrockdata(wp1), communicatedrockdata(wp2)` $\}$ from S_0. Under the independence assumption, each proposition would require only three actions (i.e., navigate, sample and communicate data), having an overall cost of 6 for S. However, we can easily see for this example, that goals are negatively interacting since we can not sample two objects unless we drop one of them, and the rover can not be at two waypoints at the same time. Ignoring these interactions for this particular example results in underestimating the real cost for supporting S. We can see that extracting effective heuristic estimators to guide state-space search is a crucial task, and one of the aims of this dissertation is to provide a flexible heuristic framework that can make state-space planning scalable to more complex planning problems. The next Chapter introduces planning graphs in the context of *Graphplan* [Blum and Furst, 1997], setting the basis for our work in domain-independent heuristics.

CHAPTER 3

Planning Graphs as a Basis for Deriving Heuristics

The efficiency of most plan synthesis algorithms and their solution quality depend highly on the informedness and admissibility of their heuristic estimators. The difficulty to improve such estimators is due to the fact that subgoals interact in complex ways. To make computation tractable, such heuristic estimators make strong assumptions about the independence of subgoals, resulting that most planners often thrash badly in problems where there are strong interactions. Furthermore, also these independence assumptions make the heuristics inadmissible affecting solution quality.

The *Graphplan* algorithm is good at dealing with problems where there are a lot of interactions between actions and subgoals providing step optimality if a solution exists. However, its main disadvantage is its backward search, which is exponential. Having to exhaust the whole search space up to the solution bearing level is a big source of inefficiency. Instead, in this chapter, we provide a way of successfully extracting heuristic estimators from *Graphplan* and use them to guide effectively *Graphplan*'s own backward search and state-space planning.

More specifically, the planning graph data structure from *Graphplan* can be seen as a compact representation of the distance metrics that estimate the cost of achieving any propo-

sition in the planning graph from the initial state. This reachability information can then be used to rank the subgoals and the actions being considered during *Graphplan*'s backward search, improving its overall efficiency [Kambhampati and Sanchez, 2000]. Furthermore, we will also show that these estimations can be combined to compute the cost of a specific state by a regression planner. This will be demonstrated through *AltAlt* [Nguyen *et al.*, 2002; Sanchez *et al.*, 2000],[1] our approach that combines the advantages of *Graphplan* and state-space search. *AltAlt* uses a Graphplan-style planner to generate a polynomial time planning data structure, which will be used to generate effective state-space search heuristics [Nguyen and Kambhampati, 2000; Nguyen *et al.*, 2002]. These heuristics are then used to control the search engine of *AltAlt*.

In the next sections, we introduce *Graphplan* and explain how distance-based heuristics are generated from its planning graph data structure. We also show how these metrics are used to improve *Graphplan*'s own backward search. Then, we extend our basic distance-based metrics to state-space search. First, we discuss the architecture of our approach *AltAlt*, and then we discuss the extensions to our heuristics to take into account complex subgoal interactions. The final section presents an empirical evaluation of our heuristic state-space planning framework.

3.1. The *Graphplan* Algorithm

One of the most successful algorithms implemented to solve classical planning problems is *Graphplan* of Blum and Furst [Blum and Furst, 1997]. The *Graphplan* algorithm can

[1] Preliminary work in *AltAlt* was done by Xuanlong Nguyen [Nguyen and Kambhampati, 2000].

be understood as a "disjunctive" version of the forward state-space planners [Kambhampati *et al.*, 1997].

The *Graphplan* algorithm alternates between two phases. A forward phase where a polynomial time data structure, called "planning graph" is incrementally expanded, and a backward phase where such structure is searched to extract a valid plan. The planning-graph (see Figure 7) consists of two alternating structures, called "proposition lists" and "action lists." Figure 7 shows a partial planning-graph structure corresponding to the rover Example 2.2.1. We start with the initial state as the zeroth level proposition list. Given a k level planning graph, the extension of structure to level $k+1$ involves introducing all actions whose preconditions are present in the k^{th} level proposition list. In addition to the actions given in the domain model, we consider a set of dummy "persist" actions (no-ops), one for each condition in the k^{th} level proposition list (represented as dashed lines in Figure 7). A "$noop_q$" action has q as its precondition and q as its effect. Once the actions are introduced, the proposition list at level $k+1$ is constructed as just the union of the effects of all the introduced actions. Planning-graph maintains the dependency links between the actions at level $k+1$ and their preconditions in level k proposition list and their effects in level $k+1$ proposition list. The planning-graph construction also involves computation and propagation of "mutex" constraints. The propagation starts at level 1, with the actions that are statically interfering with each other (i.e., their preconditions and effects are inconsistent) labeled mutex. Mutexes are then propagated from this level forward by using two simple propagation rules.

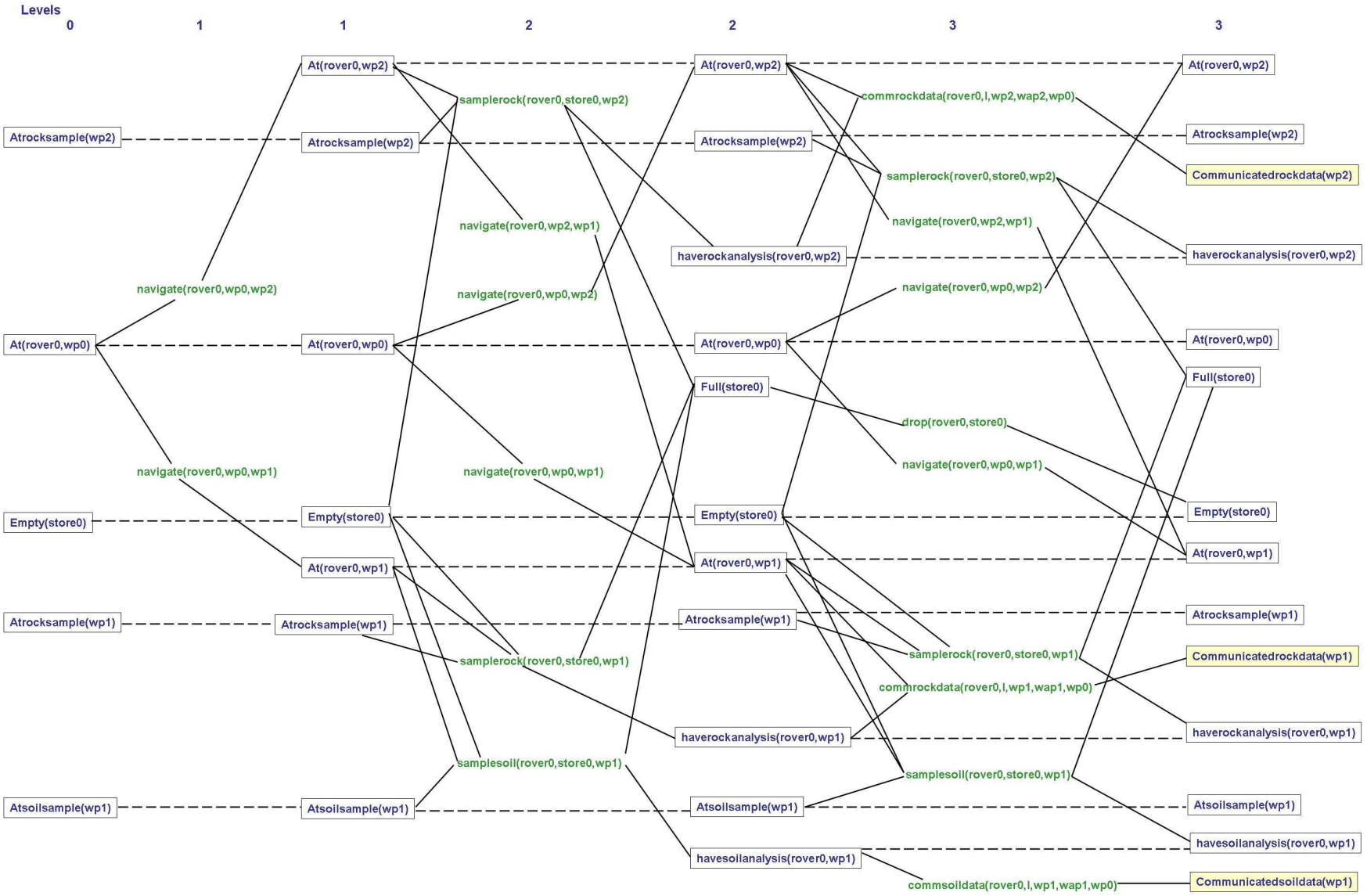


Figure 7. The Rover planning graph example. To avoid clutter, we do not show the no-ops and Mutexes.

1. Two propositions at level k are marked *mutex* if all actions at level k that support one proposition are mutex with all actions that support the second proposition.

2. Two actions at level $k+1$ are mutex if they are *statically interfering* or if one of the propositions (preconditions) supporting the first action is *mutually exclusive* with one of the propositions supporting the second action.

Notice that we have not included the mutex information in the graph of Figure 7 to avoid clutter, but we can easily see that the actions `navigate(rover0,Wp0,Wp2)` and `navigate(rover0,Wp0,Wp1)` are statically interfering, in consequence the facts `at(rover0,Wp1)` and `at(rover0,Wp2)` are mutex because all actions supporting them are mutex to each other. In our current example from Figure 7, the goals are first present at level three of the graph. However, even though it has not be shown, they are all mutexes to each other. It is not until level five in the graph when they become free mutex.

The backward phase of *Graphplan* involves checking to see if there is a subgraph from a k level planning-graph that corresponds to a valid solution to the problem. This involves starting with the propositions corresponding to goals at level k (if all the goals are not present, or if they are present but a pair of them are marked mutually exclusive, the search is abandoned right away, and planning-graph is grown another level). For each of the goal propositions, we then select an action from the level k action list that supports it, such that no two actions selected for supporting two different goals are mutually exclusive (if they are, we backtrack and try to change the selection of actions). At this point, we recursively call the same search process on the $k-1$ level planning-graph, with the preconditions of

the actions selected at level k as the goals for the $k-1$ level search. The search succeeds when we reach level 0 (corresponding to the initial state).

Graphplan's backward search phase can be seen as a CSP problem. Specifically, a dynamic constraint satisfaction problem [Mittal and Falkenhainer, 1990], where the propositions in the planning graph can be seen as CSP variables, and the actions supporting them can be seen as their values. The constraints get specified by the mutex relations [Do and Kambhampati, 2000; Lopez and Bacchus, 2003]. The next section introduces the notion of reachability in the context of *Graphplan* itself, and show how this reachability analysis can be used to develop *distance-based* estimates to improve *Graphplan*'s own backward search. Following sections will demonstrate the application of domain independent planning graph based heuristics to state-space planning.

3.2. Introducing the Notion of Heuristics and their Use in *Graphplan*

The plan synthesis algorithms explored in the previous chapter are effective in finding solutions to planning problems. However, they all suffer from the combinatorial complexity of the problems they try to solve. In consequence, one of the main directions in recent years by the planning community has been the development of heuristic search control to significantly scale up plan synthesis.

We can see in the descriptions of the algorithms presented in Section 2.3 that they traverse the search space *non-deterministically*. In order to improve their node selection, a function would be needed to select more deterministically those nodes that look more promising during search from a set of candidates. Such functions are commonly called

heuristics, and most of the times are abstract solutions to relaxed problems that are used to prioritize our original choices during search. As we can see, the main objective of a heuristic function is to guide the search of the problem in the most promising direction in order to improve the overall efficiency and scalability of the system. As we saw in Section 2.4, finding accurate heuristic estimates that take into account subgoal interactions is very important, given that even the smaller problems could be intractable in the worst case.

3.2.1. Distance-based Heuristics for *Graphplan*. As mentioned earlier, previous work has demonstrated the connections between the backward search of *Graphplan* and (dynamic) CSP problems [Mittal and Falkenhainer, 1990; Kambhampati *et al.*, 1997; Weld *et al.*, 1998]. More specifically, the propositions in the planning graph can be seen as CSP variables, while the actions supporting them can be seen as their domain of values. Constraints are then represented by the mutex relations in the graph. Given these relations, the order in which the backward search considers the (sub)goals propositions for assignment (i.e., variable ordering heuristic), and the order in which actions are chosen to support those (sub)goals (i.e., value ordering heuristic) can have a significant impact in *Graphplan*'s performance.

Our past work from [Kambhampati and Sanchez, 2000] has demonstrated that the traditional variable and value ordering heuristics from CSP literature do not work well in the context of *Graphplan*'s backward search. We then present a family of variable and value-ordering heuristics that are based on the difficulty of achieving a subgoal from the initial state. The degree of difficulty of achieving a single proposition is quantified by the

index of the *earliest level* of the planning graph in which that proposition first appears. In other words, the intuition behind the distance-based heuristics is to choose goals based on the "distance" of those goals from the initial state, where distance is interpreted as the number of actions required to go from the initial state to the goal state. It turns out that we can obtain the distances of various goal propositions through the planning graph structure. The main idea is:

> Propositions are ordered for assignment in decreasing value of their levels. Actions supporting a proposition are ordered for consideration in increasing value of their costs (see below).
>
> Where:
>
> *The level of a proposition p, lev(p) is defined as the earliest level l of the planning graph that contains p.*

These heuristics can be seen as using a "hardest to achieve goal (variable) first/easiest to support action(value) first" idea, where hardness is measure in terms of the level of the propositions. Consider the planning graph in Figure 7, the level of the top level goals is 3, while the level of `full(store0)` is 2, and that of `at(rover0,wp0)` is 0. This propagation is easy to compute in the planning graph. To support value ordering, we need to define the cost of an action a supporting a proposition p. We have three different alternatives, all of them based on the *level* information from the planning graph [Kambhampati and Sanchez, 2000]:

Mop heuristic: The cost of an action is the maximum of the cost (distance) of its preconditions. For example in Figure 7 the cost of `samplerock(rover0,store0,wp2)` is

1, since all of its preconditions appear at level 1. This heuristic is defined as:

$$Cost_{Mop}(a) = max_{p \in Prec(a)} lev(p) \tag{3.1}$$

Sum heuristic: The cost of an action is the sum of the costs of the individual propositions making up that action's precondition list, namely:

$$Cost_{Sum}(a) = \sum_{p \in Prec(a)} lev(p) \tag{3.2}$$

Consequently, in Figure 7 the cost of `samplerock(rover0,store0,wp2)` would be 3.

Level heuristic: The cost of an action is the first level at which the set of its preconditions is present in the graph without being any of them mutex with each other.[2] Following the same example from Figure 7 the cost of `samplerock(rover0,store0,wp2)` is 1 because its preconditions are non mutex at that level. The heuristic can be described as:

$$Cost_{Lev}(a) = lev(Prec(a)) \tag{3.3}$$

These simple heuristics extracted from the planning graph and the notion of *level* form the basis for building more powerful heuristics that will be applied to more complex planning frameworks discussed in this dissertation.

[2]The *Level* heuristic of an action a is just the level in the planning graph where a first occurs.

Problem	Normal GP		Mop GP		Lev GP		Sum GP		Speedup		
	Length	Time	Length	Time	Length	Time	Length	Time	Mop	Lev	Sum
BW-large-A	12/12	.008	12/12	.005	12/12	.005	12/12	.006	1.6x	1.6x	1.3x
BW-large-B	18/18	.76	18/18	.13	18/18	.13	18/18	.085	5.8x	5.8x	8.9x
BW-large-C	-	>30	28/28	1.15	28/28	1.11	-	>30	>26x	>27x	-
huge-fct	18/18	1.88	18/18	.012	18/18	.011	18/18	.024	156x	171x	78x
bw-prob04	-	>30	8/18	5.96	8/18	8	8/19	7.25	>5x	>3.7x	>4.6x
Rocket-ext-a	7/30	1.51	7/27	.89	7/27	.69	7/31	.33	1.70x	2.1x	4.5x
Rocket-ext-b	-	>30	7/29	.003	7/29	.006	7/29	.01	10000x	5000x	3000x
Att-log-a	-	>30	11/56	10.21	11/56	9.9	11/56	10.66	>3x	>3x	>2.8x
Gripper-6	11/17	.076	11/15	.002	11/15	.003	11/17	.002	38x	25x	38x
Gripper-8	-	>30	15/21	.30	15/21	.39	15/23	.32	>100x	>80	>93x
Ferry41	27/27	.66	27/27	.34	27/27	.33	27/27	.35	1.94x	2x	1.8x
Ferry-5	-	>30	33/31	.60	33/31	.61	33/31	.62	>50x	>50x	>48x
Tower-5	31/31	.67	31/31	.89	31/31	.89	31/31	.91	.75x	.75x	.73x

Table 1. Effectiveness of level heuristic in solution-bearing planning graphs. The columns titled Level GP, Mop GP and Sum GP differ in the way they order actions supporting a proposition. Mop GP considers the cost of an action to be the maximum cost of any if its preconditions. Sum GP considers the cost as the sum of the costs of the preconditions and Level GP considers the cost to be the index of the level in the planning graph where the preconditions of the action first occur and are not pair-wise mutex.

3.3. Evaluating the Effectiveness of Level-based Heuristics in *Graphplan*

We implemented the three *level-based* heuristics discussed in this chapter for *Graphplan*'s backward search, and evaluated their performance as compared to normal *Graphplan*. Our extensions were based on the version of *Graphplan* implementation bundled in the Blackbox system [Kautz and Selman, 1999], which in turn was derived from Blum & Furst's original implementation [Blum and Furst, 1997]. Table 1 shows the results on some standard benchmark problems. The columns titled "Mop GP", "Lev GP" and "Sum GP" correspond respectively to *Graphplan* armed with the $Cost_{Mop}$, $Cost_{Lev}$, and $Cost_{Sum}$ heuristics for variable and value ordering. Cpu time is shown in minutes. For our Pentium Linux machine with 256 Megabytes of RAM.[3] The table compares the effectiveness of standard *Graphplan* (with noops-first heuristic [Kambhampati and Sanchez, 2000]), and *Graphplan* with our three level-based heuristics in searching the planning graph containing minimum length solution. As can be seen, the final level search can be improved by 2 to 4 orders of magnitude with the level-based heuristics.

Empirical results demonstrate that these heuristics could speedup backward search by several orders in solution-bearing planning graphs. Our heuristics, while quite simple, are nevertheless significant in that previous attempts to devise effective variable ordering techniques for *Graphplan*'s search have not been successful.

[3]For an additional set of experiments see [Kambhampati and Sanchez, 2000].

3.4. *AltAlt*: Extending Planning Graph Based Heuristics to State-space Search

As mentioned earlier, *AltAlt* system is based on a combination of *Graphplan* and heuristic state-space search technology. The high-level architecture of *AltAlt* is shown in Figure 8. The problem specification and the action template description are first fed to a Graphplan-style planner, which constructs a planning graph for that problem in polynomial time. We use the publicly available STAN implementation [Long and Fox, 1999] for this purpose as it provides a highly memory efficient implementation of the planning graph construction phase. This planning graph structure is then fed to a heuristic extractor module that is capable of extracting a variety of effective and admissible heuristics, based on the theory that we have developed in our work [Nguyen and Kambhampati, 2000; Nguyen *et al.*, 2002], and that we will discuss in the next section. This heuristic, along with the problem specification, and the set of ground actions in the final action level of the planning graph structure are fed to a regression state search planner. The regression planner code is adapted from HSP-r [Bonet and Geffner, 1999].

To explain the operation of *AltAlt* at a more detailed level, we need to provide some further background on its various components. We shall start with the regression search module. As introduced in Section 3.4, regression search is the process of searching in the space of potential plan suffixes. The suffixes are generated by starting with the goal state and regressing it over the set of relevant action instances from the domain. The resulting states are then (non-deterministically) regressed again over relevant action instances, and this process is repeated until we reach a state (set of subgoals) which is satisfied by the

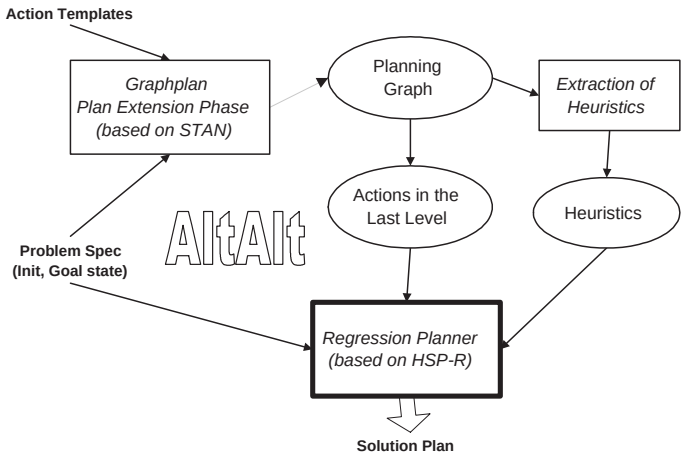


Figure 8. Architecture of *AltAlt*

initial state. As mentioned earlier, a state S in our framework is a set of (conjunction of) literals that can be seen as "subgoals" that need to be made true on the way to achieving the top level goals. An action instance a is considered relevant to a state S if the effects of a give at least one element of S and do not delete *any element* of S. The result of regressing S over a is then specified by equation 2.2 from Chapter 2. $Regress(S, a)$ represents the set of goals that still need to be achieved before the application of a, such that everything in S would have been achieved once a is applied. For each relevant action a, a separate search branch is generated, with the result of regressing S over that action as the new fringe in that branch. Search terminates with success at a node if every literal in the state corresponding to that node is present in the initial state of the problem.

The crux of controlling the regression search involves providing a heuristic function that can estimate the relative goodness of the states on the fringe of the current search tree and guide the search in most promising directions. So, to guide a regression search in the space of states, a heuristic function needs to evaluate the cost of some set S of subgoals

(comprising a regressed state), from the initial state–in terms of the length of the plan needed to achieve them from the initial state.

The search algorithm used in *AltAlt* is similar to that used in HSP-r [Bonet and Geffner, 1999]–it is a hybrid between greedy depth first and a weighted A* search. It goes depth-first as long as the heuristic cost of any of the children states is lower than that of the current state. Otherwise, the algorithm resorts to a weighted A* search to select the next node to expand. In this latter case, the evaluation function used to rank the nodes is $f(S) = g(S) + w * h(S)$, where $g(S)$ is the accumulated cost (number of actions when regressing from the goal state), $h(S)$ is the heuristic value for a given state, and w is a weight parameter set to 5.[4]

We now discuss how distance-based heuristics can be computed from the planning graphs, which, by construction, provide optimistic reachability estimates.

3.5. Extracting Effective Heuristics from the Planning Graph

Normally, the planning graph data structure supports "parallel" plans–i.e., plans where at each step more than one action may be executed simultaneously. Since we want the planning graph to provide heuristics to the regression search module of *AltAlt*, which generates sequential solutions, we first make a modification to the algorithm so that it generates a "serial planning graph." A *serial planning graph* is a planning graph in which, in addition to the normal mutex relations, every pair of non-noop actions at the same level are marked mutex. These additional action mutexes propagate to give additional

[4]For the role of w in search see [Korf, 1993]

propositional mutexes. Finally, a planning graph is said to **level-off** when there is no change in the action, proposition and mutex lists between two consecutive levels.

The critical asset of the planning graph, for our purposes, is the efficient marking an propagation of mutex constraints during the expansion phase. A mutex relation is called *static* (or "persistent") if it remains a mutex up to the level where the planning graph levels off. A mutex relation is called *dynamic* (or level specific) if it is not static.

Based on the above, the following properties can be easily verified:

1. The number of actions required to achieve a pair of propositions is no less than the index of the smallest proposition level in the planning graph in which both propositions appear without a mutex relation.

2. Any pair of propositions having a static mutex relation between them can never be true together.

3. The set of actions present in the level where the planning graph levels off contains all actions that are applicable to states reachable from the initial state.

These observations give a rough indication as how the information in the leveled planning graph can be used to guide state-space search planners. The first observation shows that the level information in the planning graph can be used to estimate the cost of achieving a set of propositions. Furthermore, the set of *dynamic* propositional mutexes help to get finer distance estimates. The second observation allows us to prove that certain world states are unreachable from the initial state pruning the search space. The third observation shows a way of extracting a finer (smaller) set of applicable actions to be considered by the regression search.

We will assume for now that given a problem, the *Graphplan* module of *AltAlt* is used to generate and expand a serial planning graph until it levels off. (As we shall see later, we can relax the requirement of growing the planning graph to level-off, if we can tolerate a graded loss of informedness of heuristics derived from the planning graph.) We will start with the notion of level of a proposition that was introduced informally before:

Definition 1 (Level)**.** *Given a proposition p, lev(p) is the index of the first level in the leveled serial planning graph in which p first appeared.*

The intuition behind this definition is that the level of a literal p in the planning graph provides a lower bound on the number of actions required to achieve p from the initial state. Following these observations, we can arrive to our first planning graph distance-based heuristic [Nguyen *et al.*, 2002; Sanchez *et al.*, 2000]:

Heuristic 1 (Max heuristic)**.** $h_{max}(S) := \max_{p \in S} lev(\{p\})$

The h_{max} heuristic is admissible, however, is not very informed as it grossly underestimates the cost of achieving a given state [Nguyen *et al.*, 2002]. Our second heuristic estimates the cost of a set of subgoals by adding up their levels:

Heuristic 2 (Sum heuristic)**.** $h_{sum}(S) := \sum_{p \in S} lev(\{p\})$

The sum heuristic is very similar to the greedy regression heuristic used in UNPOP [McDermott, 1999] and the heuristic used in the HSP planner [Bonet *et al.*, 1997]. Its main limitation is that the heuristic makes the implicit assumption that all the subgoals (elements of S) are independent. Sum heuristic is neither admissible nor particularly

informed. Specifically, since subgoals can be interacting negatively (in that achieving one winds up undoing progress made on achieving the others), the true cost of achieving a pair of subgoals may be more than the sum of the costs of achieving them individually. This makes the heuristic inadmissible. Similarly, since subgoals can be positively interacting in that achieving one winds up making indirect progress towards the achievement of the other, the true cost of achieving a set of subgoals may be lower than the sum of their individual costs. To develop more effective heuristics, we need to consider both positive and negative interactions among subgoals in a limited fashion. We start taking into account more interactions by considering the notion of level of a set of propositions:

Definition 2 (Set Level)**.** *Given a set S of propositions, $lev(S)$ is the index of the first level in the leveled serial planning graph in which all propositions in S appear and are non-mutexed with one another (If S is a singleton p, then $lev(S) = lev(p)$). If no such level exists and the planning graph has been grown to level-off then $lev(S) = \infty$. Or, $lev(S) = l + 1$, where l is the index of the last level that the planning graph has been grown to (i.e not until level-off).*

Leading us to our next heuristic:

Heuristic 3 (Set-level heuristic)**.** $h_{lev}(S) := lev(S)$

It is easy to see that set-level heuristic is *admissible*. Secondly, it can be significantly more informed than the *max heuristic*, because the max heuristic is only equivalent to the level that a single proposition first comes into the planning graph. Thirdly, a by-product of the set-level heuristic is that it is easy to compute and effective since we already have the static and dynamic mutex information from the planning graph.

However, Set-level heuristics is not perfect. It tends to be too conservative and often underestimates the real cost in domains with many independent subgoals [Nguyen *et al.*, 2002]. To overcome these limitations, two families of heuristics have been implemented that take into account subgoals interactions, "partition-k" heuristics and "adjusted-sum" heuristics [Nguyen and Kambhampati, 2000; Nguyen *et al.*, 2002].

Partition-k heuristics:

The Partition-k heuristics attempts to improve and generalize the set-level heuristic using the sum heuristic idea. Specifically, it estimates the cost of a set in terms of the costs of its partitions. When the subgoals are relatively independent, the summation of the cost of each individual gives a much better estimate, whereas the graph level value of the set tends to underestimate significantly. To avoid this problem and at the same time keep track of the interaction between subgoals, we want to partition the set S of propositions into subsets, each of which has k elements: $S = S_1 \cup S_2 ... \cup S_m$ (if k does not divide $|S|$, one subset will have less than k elements), and then apply the set-level heuristic value on each partition. Ideally, we want a partitioning such that elements within each subset S_i may be interacting with each other, but the subsets are independent (i.e non-interacting) of each other. By *interacting* we mean the two propositions form either a pair of dynamic (level-specific) or static mutex in the planning graph. These notions are formalized by the following definitions.

Definition 3. *The* ***(binary) interaction degree*** δ *between two propositions* p_1 *and* p_2 *is defined as:* $\delta(p_1, p_2) = lev(\{p_1, p_2\}) - max\{lev(p_1), lev(p_2)\}$.

When $p1$ and $p2$ are dynamic mutex, $\delta(p_1, p_2) > 0$ but $lev(\{p_1, p_2\}) < \infty$. When $p1$ and $p2$ are static mutex, $lev(\{p_1, p_2\}) = \infty$. Since we only consider propositions that are present in the planning graph, i.e $max\{lev(p_1), lev(p_2)\} < \infty$, it follows that $\delta(p_1, p_2) > 0$ as well. When $p1$ and $p2$ are neither type of mutex, $lev(\{p_1, p_2\}) = max\{lev(p_1), lev(p_2)\}$, thus $\delta(p_1, p_2) = 0$.

Definition 4. *Two propositions p and q are **interacting** with each other if and only if $\delta(p, q) > 0$. Two sets of propositions S_1 and S_2 are not interacting with each other if no proposition in S_1 is interacting with a proposition in S_2.*

Following our definitions, we are ready to state our next heuristics:

Heuristic 4 (Partition-k heuristic)**.**

$h_{part-k}(S) := \sum_{S_i} lev(S_i)$*, where $S_1, ..., S_m$ are k-sized partitions of S.*[5]

Adjusted-sum heuristics:

The second family of heuristics, called "adjusted sum" heuristics attempt to improve the h_{sum} heuristic 2 using the set-level idea 3. Specifically, it starts explicitly from the h_{sum} heuristic and then considers adding the positive and negative interactions among subgoals that can be extracted from the planning graph's level information. Since fully accounting for either type of interaction alone can be as hard as the planning problem itself, we circumvent this difficulty by using partial relaxation assumption on the subgoal interactions. Namely, we ignore one type of subgoal interaction in order to account for the other, and then combine them both together.

[5] *For a deeper analysis on this class of heuristics see [Nguyen* et al.*, 2002]*

We start with the assumption that all propositions are independent. Remember that this is a property of the h_{sum} heuristic. We assume that there are no positive interactions, but there are negative interactions among the propositions. This can be computed using the interaction degree among propositions in a set S, which is no more than an extension of Definition 3.

Definition 5. *The* ***interaction degree*** *among propositions in a set S is:*

$$\Delta(S) = h_{lev}(S) - h_{max}(S)$$

We can easily see that when there are no negative interactions among the subgoals $h_{lev}(S) = h_{max}(S)$. So, our following heuristics gets formulated as:

Heuristic 5 (Adjusted-sum heuristic)**.** $h_{adjsum}(S) := h_{sum}(S) + \Delta(S)$

As mentioned earlier, h_{sum} accounts for the cost of achieving S under the *independence* assumption, while $\Delta(S)$ accounts for the additional cost incurred by the *negative* interactions.

We can improve the heuristic estimators of $h_{adjsum}(S)$ by replacing its first term with another estimate that takes into account positive interactions. This is done by another heuristic, which we called $h_{adjsum2M}(S)$.

The basic idea of $h_{AdjSum2M}$ is to adjust the sum heuristic to take positive as well as negative interactions into account. This heuristic approximates the cost of achieving the subgoals in some set S as the sum of the cost of achieving S, while considering positive interactions and ignoring negative interactions, plus the penalty for ignoring the negative

interactions. The first component can be computed as the length of a "relaxed plan" for supporting S, which is extracted by *ignoring all the mutex relations*. The relaxed plan is computed by regressing S over an applicable action a_s, obtaining the state $S' = Regress(S, a_s)$. Getting the following recurrent relation:

$$relaxPlan(S) = 1 + relaxPlan(S') \tag{3.4}$$

This regression accounts for the positive interactions in the state S given that by subtracting the effects of a_s, any propositions that are co-achieved will not count in the cost computation. The recursive application of the last equation is bounded by the final level of the planning graph, and it will eventually reduce to a state S_0 where each proposition $q \in S_0$ is also in the initial state I.

To approximate the penalty induced by the negative interactions alone, we proceed to use the binary degree of interaction among any pair of propositions from Definition 3. We want to use the $\delta(p, q)$ values to characterize the amount of negative interactions present among the subgoals of a given set S. If all subgoals in S are pair-wise independent, clearly, all δ values will be zero, otherwise each pair of subgoals in S will have a different value. The largest such δ value among any pair of subgoals in S is used as a measure of the negative interactions present in S in the heuristic $h_{AdjSum2M}$. In summary, we have

Heuristic 6 (Adjusted heuristic 2M)**.** $h_{AdjSum2M}(S) := length(relaxPlan(S)) + max_{p,q \in S}\ \delta(p, q)$

The analysis in [Nguyen and Kambhampati, 2000; Nguyen *et al.*, 2002] shows that this is one of the more robust heuristics in terms of both solution time and quality. This is thus the default heuristic used in *AltAlt*.[6]

3.6. Controlling the Cost of Computing the Heuristics

Although planning graph construction is a polynomial time operation, it does lead to a relatively high time and space consumption in many problems. The main issues are the sheer size of the planning graph, and the cost of marking and managing mutex relations. Fortunately, however, there are several possible ways of keeping the heuristic computation cost in check. To begin with, one main reason for basing *AltAlt* on STAN rather than other *Graphplan* implementations is that STAN provides a particularly compact and efficient planning graph construction phase. In particular, as described in [Long and Fox, 1999], STAN exploits the redundancy in the planning graph and represents it using a very compact bi-level representation. Secondly, STAN uses efficient data structures to mark and manage the "mutex" relations.

While the use of STAN system reduces planning graph construction costs significantly, heuristic computation cost can still be a large fraction of the total run time. Thankfully, however, by trading off heuristic quality for reduced cost, we can aggressively limit the heuristic computation costs. Specifically, in the previous section, we discussed the extraction of heuristics from a full leveled planning graph. Since *AltAlt* does not do any search on the planning graph directly, there is no strict need to use the full leveled graph to

[6]See [Nguyen *et al.*, 2002] for an extensive presentation of these heuristics and their variations.

preserve completeness. Informally, *any subgraph* of the full leveled planning graph can be gainfully utilized as the basis for the heuristic computation. There are at least three ways of computing a smaller subset of the leveled planning graph:

1. Grow the planning graph to some length that is less than the length where it levels off. For example, we may grow the graph until the top level goals of the problem are present without any of them having mutex relations.

2. Spend only limited time on marking mutexes on the planning graph.

3. Introduce only a subset of the "applicable" actions at each level of the planning graph. For example, we can exploit the techniques such as RIFO [Kohler *et al.*, 1997] and identify a subset of the action instances in the domain that are likely to be "relevant" for solving the problem.

Any combination of the above three techniques can be used to limit the space and time resources expended on computing the planning graph. What is more, it can be shown that the admissibility and completeness characteristics of the heuristic will remain unaffected as long as we do not use the third approach. Only the informedness of the heuristic is affected. We shall see later in this chapter that in many problems the loss of informedness is more than offset by the improved time and space costs of the heuristic.

3.7. Limiting the Branching Factor of the Search Using Planning Graphs

The preceding chapters focused on the use of the planning graphs for computing the heuristics in *AltAlt*. However, from Figure 8, we can see that the planning graph is

also used to pick the action instances considered for expanding the regression search tree. The advantages of using the action instances from the planning graph are that in many domains there is a prohibitively large number of ground action instances, only a very small subset of which are actually applicable on a given state reachable from the initial state. Using all such ground actions in regression search can significantly increase the cost of node expansion (and may, on occasion, lead the search down the wrong paths). In contrast, the action instances present in the planning graph are more likely to be applicable in states reachable from the initial state.

The simplest way of picking action instances from the planning graph is to consider all action instances that are present in the final level of the planning graph. If the graph has been grown to level off, it can be proved that limiting regression search to this subset of actions is guaranteed to preserve completeness. A more aggressive selective expansion approach, that we call `PACTION`, involves the following. Suppose that we are trying to expand a state S in the regression search, then only the set of actions appearing in the action level $lev(S)$ (i.e., the index of the level of the planning graph at which the propositions in set S first appear without any pair-wise mutex relations between them) is considered to regress the state S. The intuition behind `PACTION` strategy is that the actions in level $lev(S)$ comprise the actions that are likely to achieve the subgoals of S in the most direct way from the initial state. While this strategy may in principle result in the incompleteness of the search (for example, some actions needed for the solution plan could appear much later in the graph, at levels $l > lev(S)$), we have not yet come across a single problem instance in which our strategy fails to find a solution that can be found considering the full

	STAN3.0		HSP-r		HSP2.0		AltAlt(AdjSum2M)	
Problem	Time	Length	Time	Length	Time	Length	Time	Length
gripper-15	-	-	0.12	45	0.19	57	0.31	45
gripper-20	-	-	0.35	57	0.43	73	0.84	57
gripper-25	-	-	0.60	67	0.79	83	1.57	67
gripper-30	-	-	1.07	77	1.25	93	2.83	77
tower-3	0.04	7	0.01	7	0.01	7	0.04	7
tower-5	0.21	31	5.5	31	0.04	31	0.16	31
tower-7	2.63	127	-	-	0.61	127	1.37	127
tower-9	108.85	511	-	-	14.86	511	48.45	511
8-puzzle1	37.40	31	34.47	45	0.64	59	0.69	31
8-puzzle2	35.92	30	6.07	52	0.55	48	0.74	30
8-puzzle3	0.63	20	164.27	24	0.34	34	0.19	20
8-puzzle4	4.88	25	1.35	26	0.46	42	0.41	24
aips-grid1	1.07	14	-	-	2.19	14	0.88	14
aips-grid2	-	-	-	-	14.06	26	95.98	34
mystery2	0.20	9	84.00	8	10.12	9	3.53	9
mystery3	0.13	4	4.74	4	2.49	4	0.26	4
mystery6	4.99	16	-	-	148.94	16	62.25	16
mystery9	0.12	8	4.8	8	3.57	8	0.49	8
mprime2	0.567	13	23.32	9	20.90	9	5.79	11
mprime3	1.02	6	8.31	4	5.17	4	1.67	4
mprime4	0.83	11	33.12	8	0.92	10	1.29	11
mprime7	0.418	6	-	-	-	-	1.32	6
mprime16	5.56	13	-	-	46.58	6	4.74	9
mprime27	1.90	9	-	-	45.71	7	2.67	9

Table 2. Comparing the performance of *AltAlt* with STAN, a state-of-the-art *Graphplan* system, and HSP-r, a state-of-the-art heuristic state search planner.

set of actions. As we shall see in the next section, this strategy has significant effect on the performance of *AltAlt* in some domains.

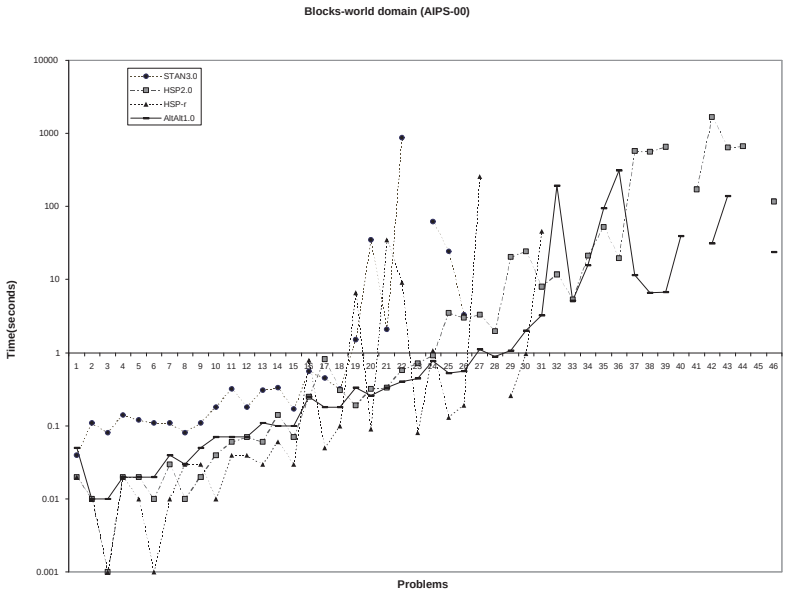
3.8. Empirical Evaluation of *AltAlt*

AltAlt planning system as described in the previous sections has been fully implemented. Its performance on many benchmark problems, as well as the test suite used in the AIPS-2000 planning competition, is remarkably robust. Our experiments suggest that *AltAlt* system is competitive with some of the best systems that participated in the AIPS-2000 competition [Bacchus, 2001]. The evaluation studies presented in this section are however

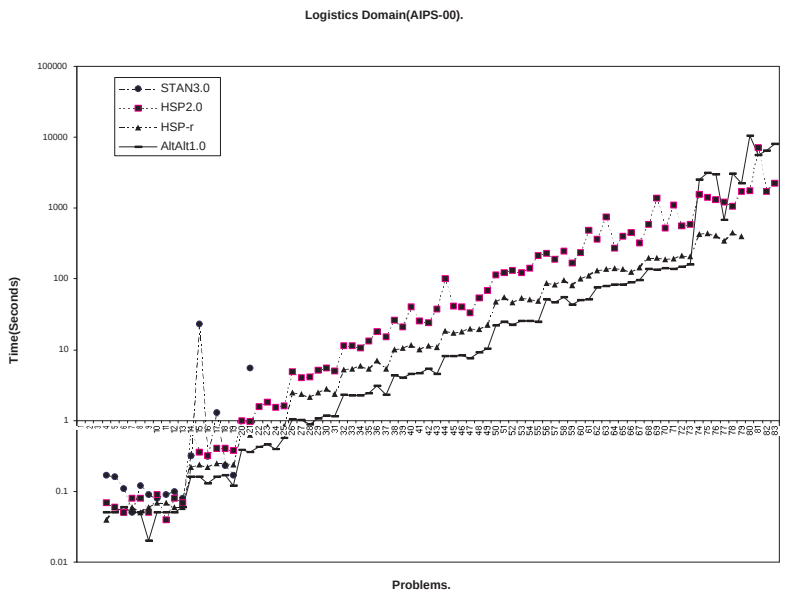
aimed at establishing two main facts: First, *AltAlt* convincingly outperforms both STAN and HSP-r systems in a variety of domains. Second, *AltAlt* is able to reduce the cost of its heuristic computation with very little negative impact on the quality of the solutions produced.

Our experiments were all done on a Linux system running on a 500 mega hertz pentium III CPU with 256 megabytes of RAM. We compared *AltAlt* with the latest versions of both STAN and HSP-r system running on the same hardware. HSP2.0 [Bonet and Geffner, 2001] is a more recent variant of the HSP-r system that opportunistically shifts between regression search (HSP-r) and progression search (HSP). We also compare *AltAlt* to HSP2.0. The problems used in our experiments come from a variety of domains, and were derived primarily from the AIPS-2000 competition suites [Bacchus, 2001], but also contain some other benchmark problems known in the literature. Unless noted otherwise, in all the experiments, *AltAlt* was run with the heuristic $h_{AdjSum2M}$, and with a planning graph grown only until the first level where top level goals are present without being mutex (see discussion in Section 3.6). Only the action instances present in the final level of the planning graph are used to expand nodes in the regression search (see Section 3.7).

Table 2 shows some statistics gathered from head-on comparisons between *AltAlt*, STAN, HSP-r and HSP2.0 across a variety of domains. For each system, the table gives the time taken to produce the solution, and the length (measured in the number of actions) of the solution produced. Dashes show problem instances that could not be solved by the corresponding system under a time limit of 10 minutes. We note that *AltAlt* demonstrates robust performance across all the domains. It *decisively outperforms* STAN and HSP-r in most of the problems, easily solving those problems that are hard for STAN as well as those

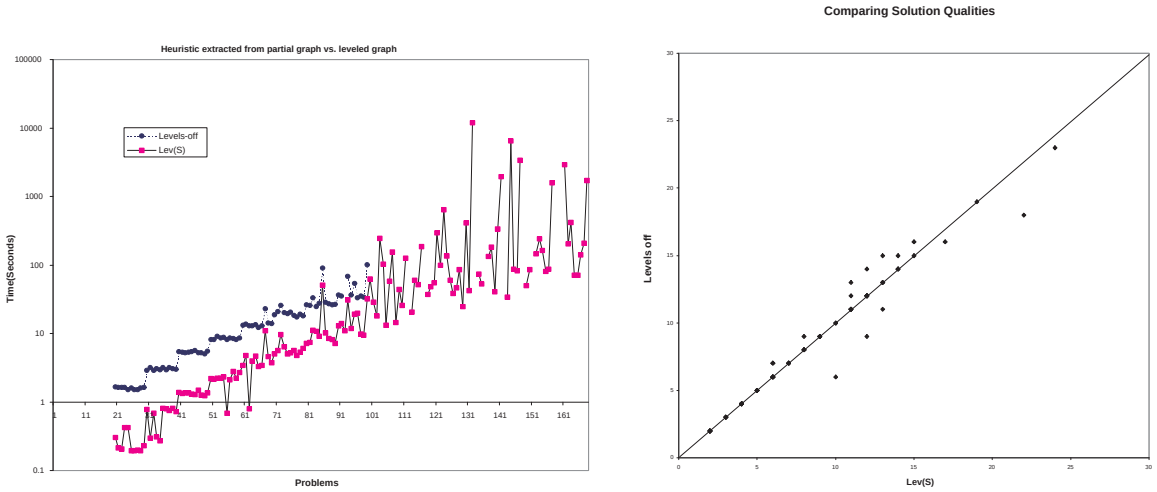


(a) Blocks World



(b) Logistics

Figure 9. Results in Blocks World and Logistics from AIPS-00



(a) Running Time

(b) Solution Quality

Figure 10. Results on trading heuristic quality for cost by extracting heuristics from partial planning graphs.

that are hard for HSP-r. We also notice that the quality of the solutions produced by *AltAlt* is as good as, or better than those produced by the other two systems in most problems. The table also shows a comparison with HSP2.0. While HSP2.0 predictably outperforms HSP-r, it is still dominated by *AltAlt*, especially in terms of solution quality.

The plots in Figure 9 compare the time performance of STAN, *AltAlt*, HSP-r and HSP2.0 in specific domains. Plot (a) summarizes the problems from blocks world and the plot (b) refers to the problems from logistics domain. We can see that *AltAlt* clearly dominates STAN. It dominates HSP2.0 and HSP-r in logistics and is very competitive with them in blocks world. Although not shown in the plots, the length of the solutions found by *AltAlt* in all these domains was as good as, or better than the rest of the systems.

Cost/Quality tradeoffs in the heuristic computation: We mentioned earlier that in all these experiments we used a partial (non-leveled) planning graph that was grown only

until all the goals are present and are non-mutex in the final level. As the discussion in Section 3.6 showed, deriving heuristics from such partial planning graphs trades cost of the heuristic computation with quality. To get an idea of how much of a hit on solution quality we are taking, we ran experiments comparing the same heuristic $h_{AdjSum2M}$ derived once from full leveled planning graph, and once from the partial planning graph stopped at the level where goals first become non-mutexed.

The plots in Figure 10 show the results of experiments with a large set of problems from the scheduling domain [Bacchus, 2001]. Plot (a) shows the total time taken for heuristic computation and search together, and Plot (b) compares the length of the solution found for both strategies. We can see very clearly that if we insist on full leveled planning graph, we are unable to solve problems beyond 81, while the heuristic derived from the partial planning graph scales all the way to 161 problems. As expected, the time taken by the partial planning graph strategy is significantly lower. Plot b shows that even on the problems that are solved by both strategies, we do not incur any appreciable loss of solution quality because of the use of partial planning graph. The few points below the diagonal correspond to the problem instances on which the plans generated with the heuristic derived from the partial planning graph were longer than those generated with heuristic derived from the full leveled planning graph. This validates our contention in Section 3.6 that the heuristic computation cost can be kept within limits without an appreciable loss in efficiency of search or the quality of the solution. It should be mentioned here that the planning graph computation cost depends a lot upon domains. In domains such as Towers of hanoi, where there are very few irrelevant actions, the full and partial planning graph strategies are almost indistinguishable in terms of cost. In contrast, domains such as grid world and scheduling world incur significantly

higher planning graph construction costs, and thus benefit more readily by the use of partial planning graphs.[7]

[7]For a most comprehensive set of experiments see [Nguyen *et al.*, 2002].

CHAPTER 4

Generating Parallel Plans Online with State-space Search

We have introduced heuristic state-space search planning in the first chapters, and we have seen that it is one of the most efficient planning frameworks for solving large deterministic planning problems [Bonet *et al.*, 1997; Bonet and Geffner, 1999; Bacchus, 2001]. Despite its near dominance, planners based on this framework can not generate efficiently "parallel plans" [Haslum and Geffner, 2000]. Parallel plans allow concurrent execution of multiple actions in each time step. Such concurrency is likely to be more important as we progress to temporal domains. While disjunctive planners such as *Graphplan* [Blum and Furst, 1997] SATPLAN [Kautz and Selman, 1992] and GP-CSP [Do and Kambhampati, 2000] seem to have no trouble generating such parallel plans, planners that search in the space of states are overwhelmed by this task. The main reason for this is that straightforward methods for generation of parallel plans would involve progression or regression over sets of actions. This increases the branching factor of the search space exponentially. Given n actions, the branching factor of a simple progression or regression search is bounded by n, while that of progression or regression search for parallel plans will be bounded by 2^n.

The inability of state search planners in producing parallel plans has been noted in the literature previously. Past attempts to overcome this limitation have not been very successful. Indeed, in [Haslum and Geffner, 2000] Haslum and Geffner consider the problem of generating parallel plans using regression search in the space of states. They notice that the resulting planner, HSP*p, scales significantly worse than *Graphplan*. In [Haslum and Geffner, 2001], they also present TP4, which in addition to being aimed at actions with durations, also improves the branching scheme of HSP*p, by making it incremental along the lines of *Graphplan*. Empirical studies reported in [Haslum and Geffner, 2001] however indicate that even this new approach, unfortunately, scales quite poorly compared to *Graphplan* variants.

Given that the only way of generating efficiently optimal parallel plans involves using disjunctive planners, we might want to consider ways of generating near-optimal parallel plans using state-space search planners. An alternative, that we explore in this dissertation, involves incremental and greedy online parallelization. Specifically, we have developed a planner called $AltAlt^p$, which is a variant of the *AltAlt* planner [Nguyen *et al.*, 2002; Sanchez *et al.*, 2000] that uses planning graph heuristics and a compression algorithm to generate parallel plans. The idea is to search in the space of regression over single actions. Once the most promising single action to regress is selected, $AltAlt^p$ then attempts to parallelize ("fatten") the selected search branch with other independent actions. This parallelization is done in a greedy incremental fashion based on our planning graph heuristics. Actions are considered for addition to the current search branch based on the heuristic cost of the subgoals they promise to achieve. The parallelization continues to the next step only if the state resulting from the addition of the new action has a better heuristic cost. The sub-

optimality introduced by the greedy nature of the parallelization is offset to some extent by a plan-compression procedure called "*PushUp*" that tries to rearrange the evolving parallel plans by pushing up actions to higher levels in the search branch (i.e. later stages of execution) in the plan.

Despite the seeming simplicity of our approach, we will show that it is quite robust in practice. In fact, our experimental comparison with five competing planners–STAN [Long and Fox, 1999], LPG [Gerevini and Serina, 2002], Blackbox [Kautz and Selman, 1992], SAPA [Do and Kambhampati, 2001] and TP4 [Haslum and Geffner, 2001]– shows that $AltAlt^p$ is a viable and scalable alternative for generating parallel plans in several domains. For many problems, $AltAlt^p$ is able to generate parallel plans that are close to optimal in step length. It also seems to retain the efficiency advantages of heuristic state search over disjunctive planners, producing plans in a fraction of the time taken by the disjunctive planners in many cases.

In the rest of this chapter, we discuss the implementation and evaluation of our approach to generate parallel plans with $AltAlt^p$. Section 4.1 introduces alternative approaches to generate parallel plans, including post-processing the sequential plans given as input. Then, we present the architecture of our approach and our algorithm in Sections 4.2 and 4.3. The plan compression procedure used to improve the quality of $AltAlt^p$'s solutions is described in Section 4.4. We finish this chapter with an empirical evaluation of our system.

4.1. Preliminaries: Alternative Approaches and the Role of Post-processing

As mentioned earlier disjunctive planners (e.g. *Graphplan*) do not have problems in generating parallel plans. For example, *Graphplan* builds a planning graph of $k - length$ and starts searching for a solution in it. If there is no solution, then the graph gets expanded one more layer. The solution extraction phase searches for a plan by considering each of the n subgoals in turn, selecting for each of them a correspondent action. So, *Graphplan* could include at most n supporting actions in a single step, allowing concurrency. The problem with this iterative approach is that it is exponential, so we have to look into some alternative techniques.

Another way of producing parallel plans that has been studied previously in the literature is to post-process sequential plans. Techniques to optimize plans according to different criteria (e.g. execution time, quality, etc) has been done offline. The post-processing computation of a given plan to maximize its parallelism has been discussed in [Backstrom, 1998].

Reordering and de-ordering techniques are used to maximize the parallelism of the plan. In de-ordering techniques ordering relations can only be removed, not added. In reordering, arbitrary modifications to the plan are allowed. In the general case this problem is NP-Hard and it is difficult to approximate [Backstrom, 1998]. Furthermore, post-processing techniques are just concerned with modifying the order between the existing actions of a given plan, which may result in plans that are not close to optimal parallel plans if the plan given as input does not have any concurrency flexibility.

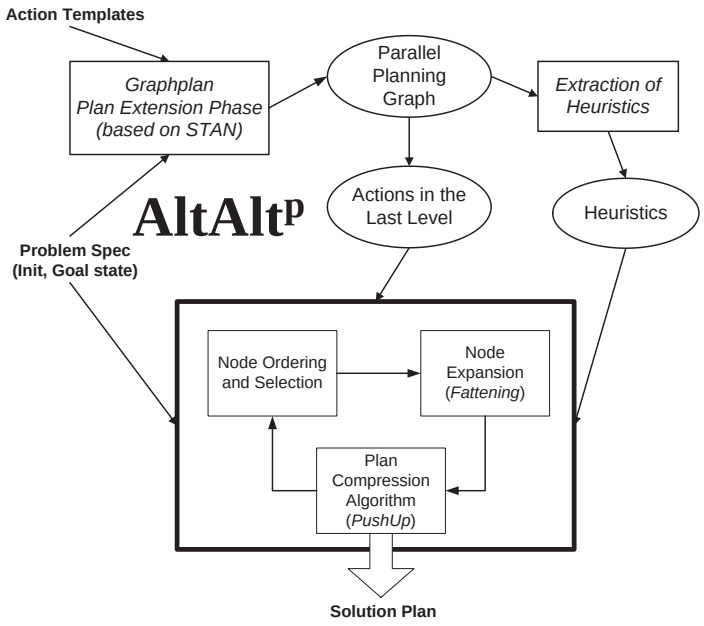


Figure 11. Architecture of *AltAlt*^{p}

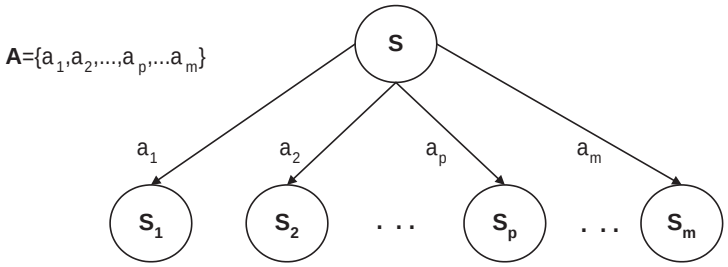


Figure 12. *AltAlt*^{p} notation

4.2. Introducing *AltAlt*^{p}, its Architecture and Heuristics

In order to avoid search directly in the whole space of parallel plans, *AltAlt*^{p} uses a greedy depth-first approach that makes use of its heuristics to regress single actions, and incrementally parallelizes the partial plan at each step, rearranging the partial plan later if necessary.

The high level architecture of $AltAlt^p$ is shown in Figure 11. Notice that the heuristic extraction phase of $AltAlt^p$ is very similar to that of $AltAlt$, but with one important modification. In contrast to $AltAlt$ which uses a "serial" planning graph as the basis for its heuristic (see Section 3.5), $AltAlt^p$ uses the standard "parallel" planning graph. This makes sense given that $AltAlt^p$ is interested in parallel plans while $AltAlt$ was aimed at generating sequential plans. The regression state-space search engine for $AltAlt^p$ is also different from the search module in $AltAlt$. $AltAlt^p$ augments the search engine of $AltAlt$ with (1) a fattening step and (2) a plan compression procedure ($PushUp$).

The general idea in $AltAlt^p$ is to select a fringe action a_p from among those actions A used to regress a particular state S during any stage of the search (see Figure 12). Then, the pivot branch given by the action a_p is "fattened" by adding more actions from A, generating a new state that is a consequence of regression over multiple parallel actions. The candidate actions used for fattening the pivot branch must (a) come from the sibling branches of the pivot branch, (b) be pairwise independent with all the other actions currently in the pivot branch and (c) lead to better heuristic estimates. We use the standard definition of action independence: two actions a_1 and a_2 are considered independent if the state S' resulting after regressing both actions simultaneously is the same as that obtained by applying a_1 and a_2 sequentially with any of their possible linearizations. A sufficient condition for this is

$$((prec(a_1) \cup eff(a_1)) \cap (prec(a_2) \cup eff(a_2))) = \emptyset$$

We now discuss in the next sections the details of the two main phases of *AltAlt*^{p} used to generate parallel plans online, the fattening procedure and the plan compression algorithm.

4.3. Selecting and Fattening a Search Branch in *AltAlt*^{p}

The first step in the *AltAlt*^{p} search engine is to select and parallelize a branch in the search tree. Figure 13 shows the steps of the fattening procedure. The procedure first identifies the set of regressable actions A for the current node S, and regresses each of them computing the new children states. Next, the action leading to the child state with the lowest heuristic cost among the new children is selected as the pivot action a_p, and the corresponding branch becomes the pivot branch.

The heuristic cost of the states is computed with the $h_{adjsum2M}$ heuristic 6 from Chapter 3, based on a "parallel" planning graph. Based on the discussion on that section, we compute the $\delta(p, q)$ values, which in turn depend on the $level(p), level(q)$ and $level(p, q)$ in terms of the levels of a parallel planning graph rather than a serial planning graph. It is easy to show that the level of a set of conditions on the parallel planning graph will be less than or equal to the level on the serial planning graph. The length of the relaxed plan is still computed in terms of number of actions.

The search algorithm used in *AltAlt*^{p} is similar to that used in *AltAlt* [Nguyen *et al.*, 2002; Sanchez *et al.*, 2000], which has been already introduced in Chapter 3. But this time, we slightly modify the evaluation function used to rank the nodes ($f(S) = g(S) + w * h(S)$). In *AltAlt*^{p}, $g(S)$ is the length of the current partial plan in terms of its number of steps,

```
parexpand(S)
    A ← get set of applicable actions for current state S
    forall a_i ∈ A
       S_i ← Regress(S,a_i)
       CHILDREN(S) ← CHILDREN(S) + S_i
    S_p ← The state among Children(S) with minimum
    h_adjsum2M value
    a_p ← the action that regresses to S_p from S
                        /**Fattening process
    O ← { a_p }
    forall g ∈ S ranked in the decreasing order of level(g)
          Find an action a_g ∈ A supporting g such that a_g ∉ O
          and a_i is pairwise independent with each action in O.
          If there are multiple such actions, pick the one that has
          minimum h_adjsum(Regress(S, O + a_g)) among all a_g ∈ A
          If h_adjsum2M(S, O + a_i) < h_adjsum2M(S, O)
             O ← O + a_g
    S_par ← Regress(S, O)
    CHILDREN(S) ← CHILDREN(S) + S_par
    return CHILDREN
END;
```

Figure 13. Node expansion procedure

where each step may have multiple concurrent actions. $h(S)$ is our estimated cost given by the heuristic function based on a parallel planning graph, and w remains the same.

In case of a tie in selecting the pivot branch, i.e., more than one branch leads to a state with the lowest heuristic cost, we break the tie by choosing the action that supports subgoals that are harder to achieve. Here, the hardness of a literal l is measured in terms of the level in the planning graph at which l first appears. The standard rationale for this decision (c.f. [Kambhampati and Sanchez, 2000]) is that we want to fail faster by considering the most difficult subgoals first. We have an additional justification in our case, we also know that a subgoal with a higher level value requires more steps and actions for its achievement because it appeared later into the planning graph. So, by supporting it

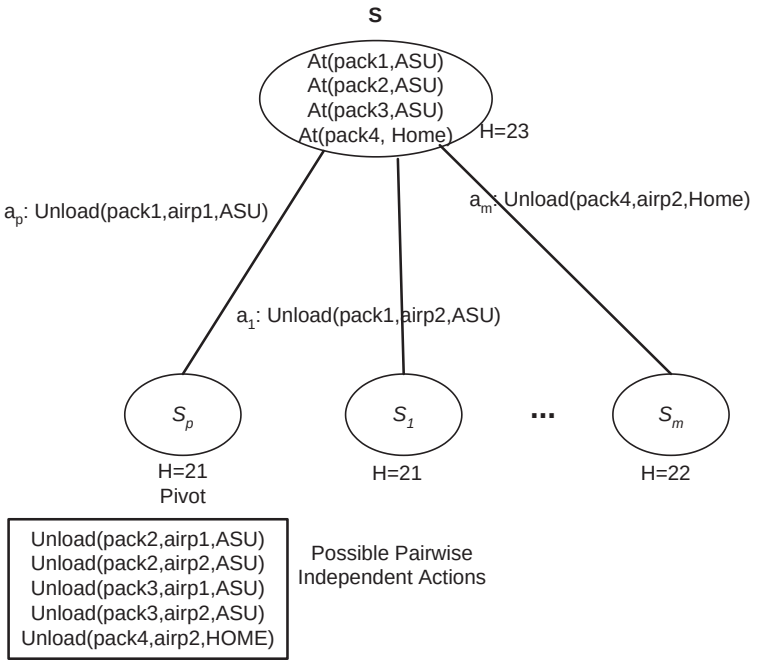


Figure 14. After the regression of a state, we can identify the *Pivot* and the related set of pairwise independent actions.

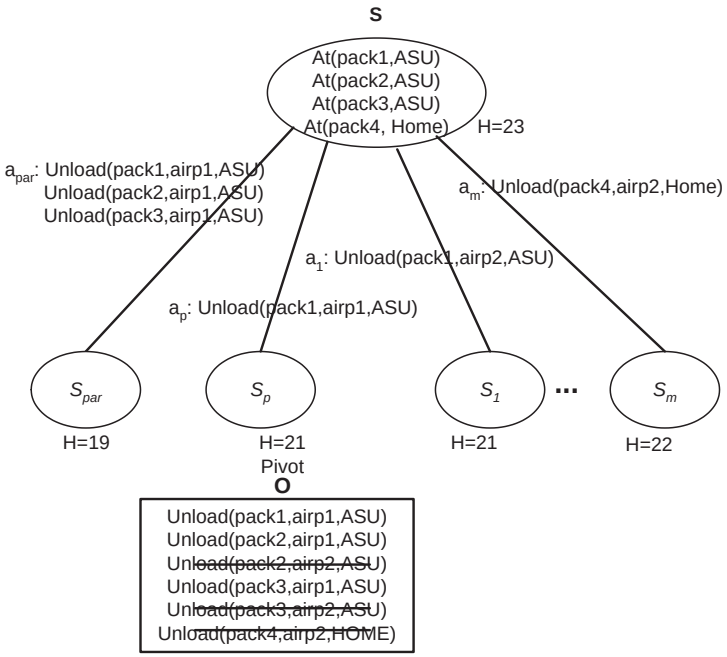


Figure 15. S_{par} is the result of incrementally fattening the *Pivot* branch with the pairwise independent actions in O

first, we may be able to achieve other easier subgoals along the way and thereby reduce the number of parallel steps in our partial plan.

Fattening the Pivot Branch: Next the procedure needs to decide which subset $O \subseteq A$ of the sibling actions of the pivot action a_p will be used to fatten the pivot branch. The obvious first idea would be to fatten the pivot branch maximally by adding all pairwise independent actions found during that search stage. The problem with this idea is that it may add redundant and heuristically inferior actions to the branch, and satisfying their preconditions may lead to an increase of the number of parallel steps.

So, in order to avoid fattening the pivot branch with such irrelevant actions, before adding any action a to O, we *require* that the heuristic cost of the state S' that results from regressing S over $O + a$ is strictly lower than that of S. This is in addition to the requirement that a is pairwise independent with the current set of actions in O. This simple check also ensures that we do not add more than one action for supporting the same set of subgoals in S.

The overall procedure for fattening the pivot branch thus involves picking the next hardest subgoal g in S (with hardness measured in terms of the level of the subgoal in the planning graph), and finding the action $a_g \in A$ achieving g, which is pair-wise independent of all actions in O and which, when added to O and used to regress S, leads to a state S' with the lowest heuristic cost. Once found, a_g is then added to O, and the procedure is repeated. If there is more than one action that can be a_g, then we break ties by considering the degree of overlap between the preconditions of action a_g and the set of actions currently in O.

Definition 6. *The **degree of precondition overlap** between an action a and the set of actions chosen O is:*

$$|prec(a) \cap \{\cup_{o \in O} prec(o)\}|$$

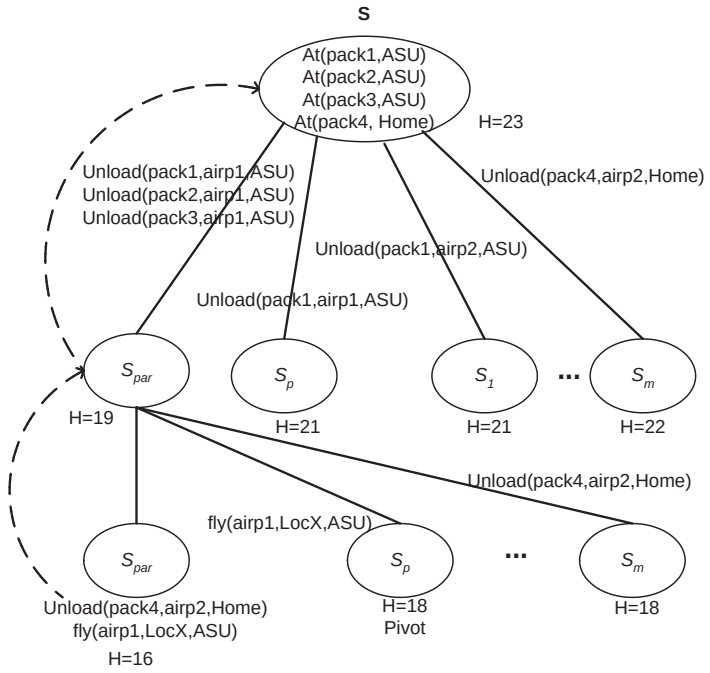
The action a with higher degree of overlap is preferred as this will reduce the amount of additional work we will need to do to establish its preconditions. Notice that because of the fattening process, a search node may have multiple actions leading to it from its parent, and multiple actions leading from it to each of its children.

Example 4.3

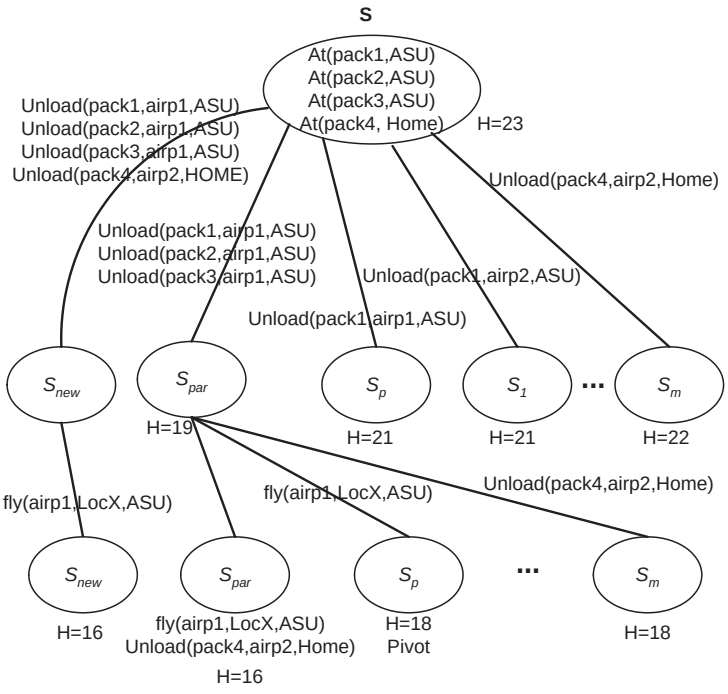
Figure 14 illustrates the use of this node expansion procedure for a problem from the logistics domain [Bacchus, 2001]. In this example we have four packages `pack1, pack2, pack3` and `pack4`. Our goal is to place the first three of them at `ASU` and the remaining one at `home`. There are two planes `airp1` and `airp2` to carry out the plans. The figure shows the first level of the search after S has been regressed. It also shows the pivot action a_p given by `unload(pack1,airp1,ASU)`, and a candidate set of pairwise independent actions with respect to a_p. Finally, we can see on Figure 15 the generation of the parallel branch. Notice that each node can be seen as a partial regressed plan. As described in the paragraphs above, only actions regressing to lower heuristics estimates are considered in a_{par} to fatten the pivot branch. Furthermore, we can also see that we have preferred actions using the plane `airp1`, since they a higher degree of overlap with the pivot action a_p.

```
pushUP(S)
    A_s ← get actions leading to S
    forall a ∈ A_s
        x ← 0
        S_x ← get parent node of S
                /** Getting highest ancestor for each action
        Loop
            A_x ← get actions leading to S_x
            If (parallel(a, A_x))
                x ← x + 1
                S_x ← get parent node of S_{x-1}
            Else
                a_j ← get action conflicting with a from A_x
                If (Secondary Optimizations)
                    Remove a and a_j from branch
                    Include a_new if necessary
                Else
                    A_{x-1} ← A_{x-1} + a
                A_s ← A_s − a
            break
        End Loop
                /**Adjusting the partial plan
    S_x ← get highest ancestor x in history
    createNewBranchFrom(S_x)
    while x > 0
        S_new ← regress S_x with A_{x-1}
        S_x ← S_new
        x ← x − 1
END;
```

Figure 16. *PushUp* procedure



(a) Finding the highest ancestor node to which an action can be pushed up.



(b) The *PushUp* procedure generates a new search branch.

Figure 17. Rearranging of the partial plan

4.4. Compressing Partial Plans to Improve Parallelism

The fattening procedure is greedy, since it insists that the state resulting after fattening have a strictly better heuristic value. While useful in avoiding the addition of irrelevant actions to the plan, this procedure can also sometimes preclude actions that are ultimately relevant but were discarded because the heuristic is not perfect. These actions may then become part of the plan at later stages during search (i.e., earlier parts of the execution of eventual solution plan; since we are searching in the space of plan suffixes). When this happens, the parallel length of the solution plan is likely to be worsened, since more steps that may be needed to support the preconditions of such actions would be forced to come at even later stages of search (earlier parts of the plan). Had the action been allowed to come into the partial plan earlier in the search (i.e., closer to the end of the eventual solution plan), its preconditions could probably have been achieved in parallel to the other subgoals in the plan, thus improving the number of steps.

In order to offset this negative effect of greediness, $AltAlt^p$ re-arranges the partial plan to promote such actions higher up the search branch (i.e., later parts of the execution of the eventual solution plan). Specifically, before expanding a given node S, $AltAlt^p$ checks to see if any of the actions in A_s leading to S from its parent node (i.e., Figure 15 shows that A_{par} leads to S_{par}) can be pushed up to higher levels in the search branch. This online re-arrangement of the plan is done by the *PushUp* procedure, which is shown in Figure 16. The *PushUp* procedure is called each time before a node gets expanded, and it will try to compress the partial plan. For each of the actions $a \in A_s$ we find the highest ancestor node S_x of S in the search branch to which the action can be applied (i.e., it gives some literal in

S_x without deleting any other literals in S_x, and it is pairwise independent of all the actions A_x currently leading out of S_x, in other words the condition $parallel(a, A_x)$ is satisfied). Once S_x is found, a is then removed from the set of actions A_s leading to S and introduced into the set of actions leading out of S_x (to its child in the current search branch). Next, the states in the search branch below S_x are adjusted to reflect this change. The adjustment involves recomputing the regressions of all the search nodes below S_x. At first glance, this might seem like a transformation of questionable utility since the preconditions of a (and their regressions) just become part of the descendants of S_x, and this does not necessarily reduce the length of the plan. We however expect a length reduction because actions supporting the preconditions of a will get "pushed up" eventually during later expansions.

Rather than doctor the existing branch, in the current implementation, we just add a new branch below S_x that reflects the changes made by the *PushUp* procedure.[1] The new branch then becomes the active search branch, and its leaf node is expanded next.

The *PushUp* procedure, as described above, is not expensive as it only affects the current search branch, and the only operations involved are recomputing the regressions in the branch. Of course, it is possible to be more aggressive in manipulating the search branch. For example, after applying an action a to its ancestor S_x the set of literals in the child state, say S_{new} changes, and thus additional actions may become relevant for expanding S_{new}. In principle, we could re-expand S_{new} in light of the new information. We decided not to go with the re-expansion option, as it typically does not seem to be worth the cost. In the next section, we do compare our default version of *PushUp* procedure with

[1]Because of the way our data structures are set up, adding a new search branch turned out to be a more robust option than manipulating the existing one.

a variant that re-expands all nodes in the search branch, and the results of those studies support our decision to avoid re-expansion. Finally, although we introduced the *PushUp* procedure as an add-on to the fattening step, it can also be used independent of the latter, in which case the net effect would be an incremental parallelization of a sequential plan.

Example 4.4

In Figure 17(a), we have two actions leading to the node S_{par} (at depth two), these two actions are `Unload(pack4,airp2,Home)` and `fly(airp1,LocX,ASU)`. So, before expanding S_{par} we check if any of the two actions leading to it can be pushed up. While the second action is not pushable since it interacts with the actions in its ancestor node, the first one is. We find the highest ancestor in the partial plan that interacts with our pushable action. In our example the root node is such an ancestor. So, we insert our pushable action `Unload(pack4,airp2,Home)` directly below the root node. We then re-adjust the state S_{par} to S_{new} at depth 1, as shown in Figure 17(b) adding a new branch, and reflecting the changes in the states below. Notice that we have not re-expanded the state S_{new} at depth 1, we have only made the adjustments to the partial plan using the actions already presented in the search trace.[2]

4.5. Results from Parallel Planning

We have implemented *AltAlt*^{p} on top of *AltAlt*. We have tested our implementation on a suite of problems that were used in the 2000 and 2002 AIPS competition [Bacchus,

[2]Instead, the aggressive *PushUp* modification would expand S_{new} at depth 1, generating similar states to those generated by the expansion of S_{par} at the same depth.

2001; Long and Fox, 2003], as well as other benchmark problems [McDermott, 2000]. Our experiments are broadly divided into three sets, each aimed at comparing the performance of $AltAlt^p$ under different scenarios:

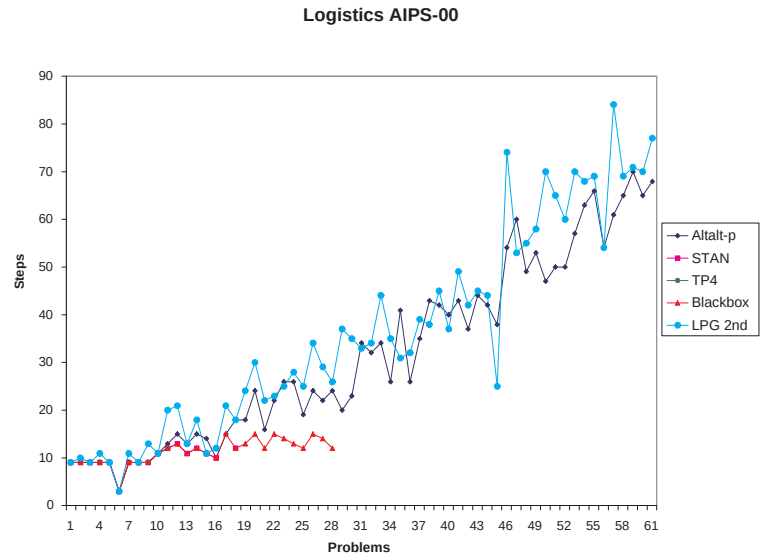
1. Comparing the performance of $AltAlt^p$ to other planning systems capable of producing parallel plans.

2. Comparing our incremental parallelization technique to $AltAlt$ + Post-Processing.

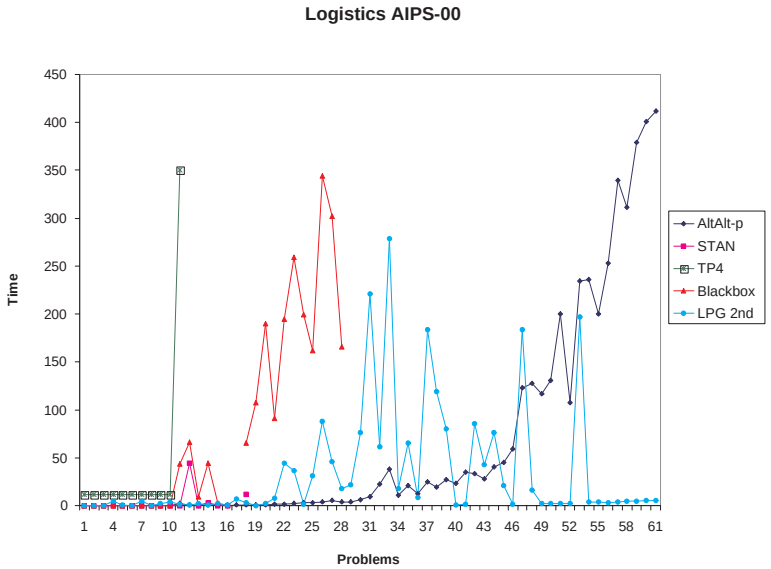
3. Ablation studies to analyze the effect of the different parts of the $AltAlt^p$ approach on its overall performance.

Our experiments were all done on a Sun Blade-100 workstation, running SunOS 5.8 with 1GB RAM. Unless noted otherwise, $AltAlt^p$ was run with the $h_{adjsum2M}$ heuristics described in section 3.5 of this proposal, and with a parallel planning graph grown until the first level where the top goals are present without being mutex. All times are in seconds.

4.5.1. Comparing $AltAlt^p$ with Competing Approaches. In the first set of experiments we have compared the performance of our planner with the results obtained by running STAN [Long and Fox, 1999], Blackbox [Kautz and Selman, 1999], TP4 [Haslum and Geffner, 2001], LPG [Gerevini and Serina, 2002] and SAPA [Do and Kambhampati, 2001]. Unless noted otherwise, every planner has been run with its default settings. Some of the planners could not be run in some domains due to parsing problems or memory allocation errors. In such cases, we just omit that planner from consideration for those particular domains.



(a)



(b)

Figure 18. Performance on Logistics (AIPS-00)

4.5.1.1. *Planners used in the Comparison Studies.* As mentioned before, STAN is an optimized version of the *Graphplan* algorithm that reasons with invariants and symmetries to reduce the size of the search space. Blackbox is also based on the *Graphplan* algorithm but it works by converting planning problems specified in STRIPS [Fikes and Nilsson, 1971] notation into boolean satisfiability problems, solving them using a SAT solver (the version we used defaults to SATZ).[3] LPG [Gerevini and Serina, 2002] was judged the best performing planner at the 3rd International Planning Competition [Long and Fox, 2003], and it is a planner based on planning graphs and local search inspired by the Walksat approach [Kautz and Cohen, 1994]. LPG was run with its default heuristics and settings. Since LPG employs an iterative improvement algorithm, the quality of the plans produced by it can be improved by running it for multiple iterations (thus increasing the running time). To make the comparisons meaningful, we decided to run LPG for two iterations (n=2), since beyond that, the running time of LPG was generally worse than that of $AltAlt^p$. Finally, we have also chosen two metric temporal planners, which are able to represent parallel plans because of their representation of time and durative actions. We consider TP4 [Haslum and Geffner, 2001], and the last planner in our list is SAPA [Do and Kambhampati, 2001]. SAPA is a powerful domain-independent heuristic forward chaining planner for metric temporal domains that employs distance-based heuristics [Kambhampati and Sanchez, 2000] to control its search.

Logistics: The plots corresponding to the Logistics domain from [Bacchus, 2001] are shown in Figure 18. For some of the most difficult problems $AltAlt^p$ outputs lower quality solutions

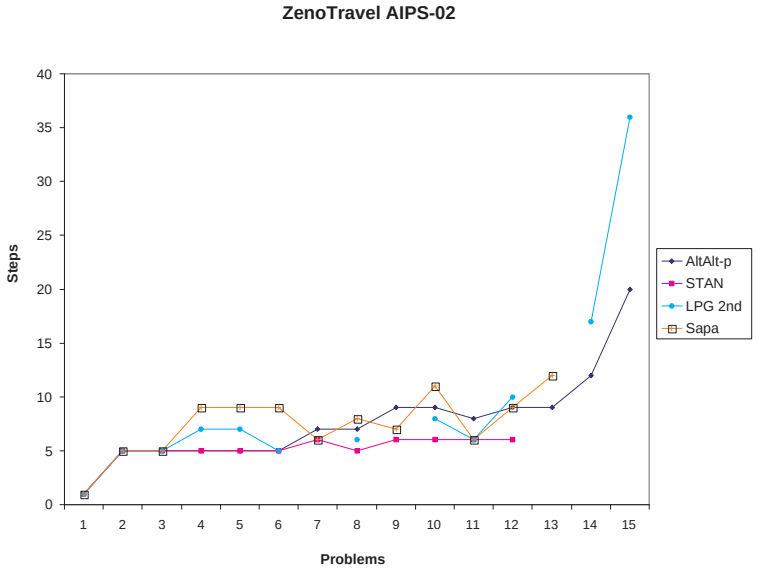
[3] We have not chosen IPP [Koehler, 1999], which is also an optimized *Graphplan* planning system because results in [Haslum and Geffner, 2001] show that it is already less efficient than STAN.

than the optimal approaches. However, only *AltAlt*^{p} and LPG are able to scale up to more complex problems, and we can easily see that *AltAlt*^{p} provides better quality solutions than LPG. *AltAlt*^{p} also seems to be more efficient than any of the other approaches. The LPG solutions for problems 49 to 61 are obtained doing only one iteration, since LPG was not able to complete the second iteration in a reasonable amount of time. This explains the low time taken for LPG, but also the lower quality of its solutions.

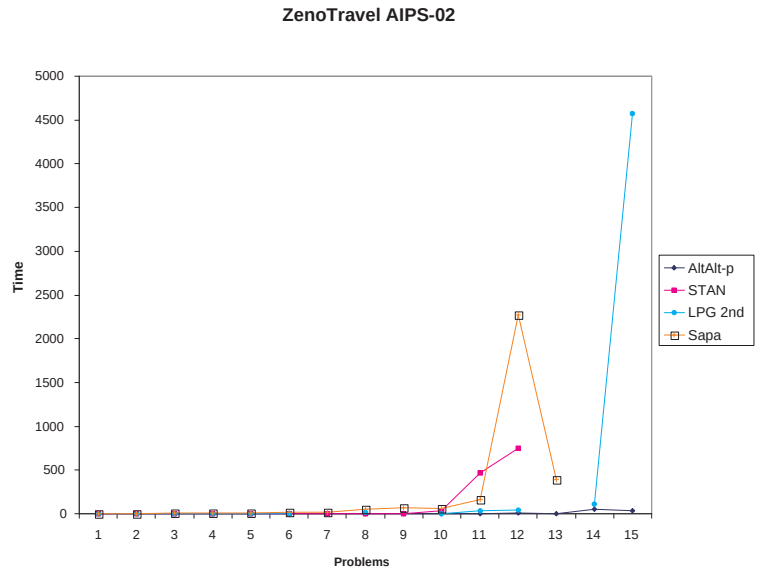
Zeno-Travel: Only *AltAlt*^{p}, SAPA, and LPG are able to solve most of the problems in this domain. *AltAlt*^{p} solves them very efficiently (Figure 19(b)) providing very good solution quality (Figure 19(a)) compared to the temporal metric planners.

Summary: In summary, we note that *AltAlt*^{p} is second only to the Blackbox algorithm in the problems that this optimal planner can solve in the Logistics domain. However, it scales up along with LPG to bigger size problems, returning very good step-length quality plans. In the ZenoTravel domain *AltAlt*^{p} is very efficient returning very good solutions. TP4, the only other heuristic state-space search regression planner capable of producing parallel plans is not able to scale up in most of the domains. SAPA, a heuristic search progression planner, while competitive, is still outperformed by *AltAlt*^{p} in planning time and solution quality.

4.5.2. Comparison to Post-Processing Approaches. As we mentioned earlier (see Section 4.1), one way of producing parallel plans that has been studied previously in the literature is to post-process sequential plans [Backstrom, 1998]. To compare online parallelization to post-processing, we have implemented Backstrom's "Minimal De-ordering Algorithm" [Backstrom, 1998], and used it to post-process the sequential plans produced

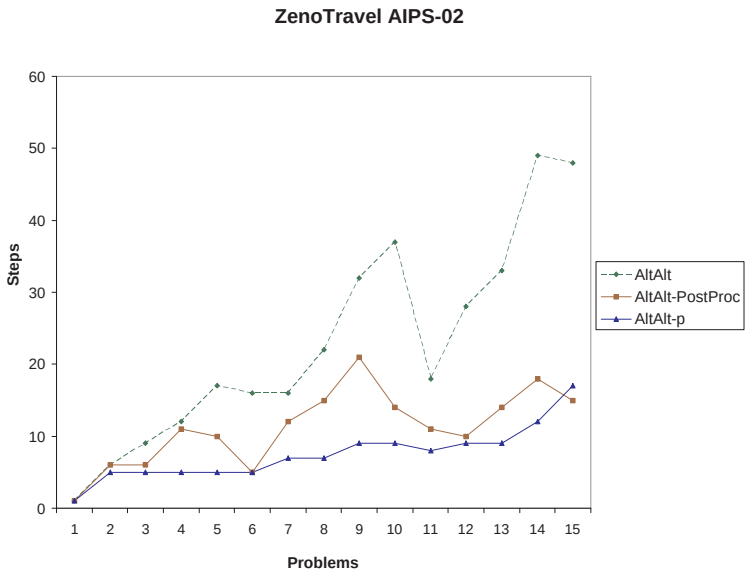


(a)

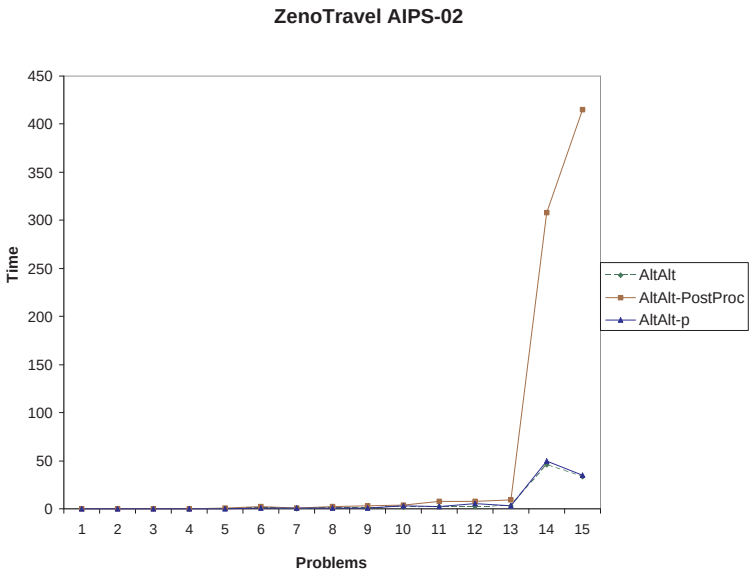


(b)

Figure 19. Performance on ZenoTravel (AIPS-02)



(a)



(b)

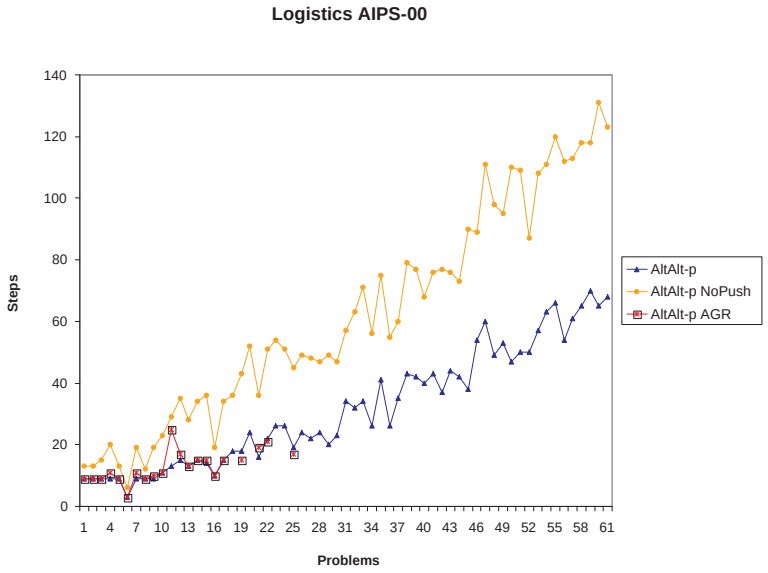
Figure 20. *AltAlt* and Post-Processing *vs.* *AltAlt*^{p} (Zenotravel domain)

by *AltAlt* (running with its default heuristic $h_{AdjSum2M}$ using a serial planning graph). In this section we will compare our online parallelization procedure to this offline method.

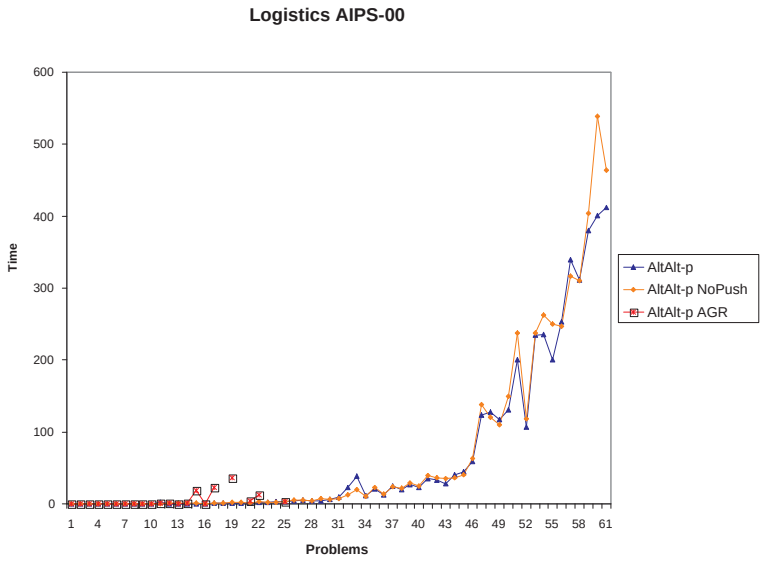
Figure 20 shows some experiments in the ZenoTravel domain [Long and Fox, 2003]. As expected, the original *AltAlt* has the longest plans since it allows only one action per time step. The plot shows that post-processing techniques do help in reducing the makespan of the plans generated by *AltAlt*. However, we also notice that *AltAlt*^{p} outputs plans with better makespan than either *AltAlt* or *AltAlt* followed by post-processing. This shows that online parallelization is a better approach than post-processing sequential plans. Moreover, the plot in Figure 20(b) shows that the time taken by *AltAlt*^{p} is largely comparable to that taken by the other two approaches. In fact, there is not much additional cost overhead in our procedure.

4.5.3. Ablation Studies. This section attempts to analyze the impact of the different parts of *AltAlt*^{p} on its performance.

Utility of the *PushUp* Procedure: Figure 21 shows the effects of running *AltAlt*^{p} with and without the *PushUp* procedure (but with the fattening procedure), as well as running it with a more aggressive version of *PushUp*, which as described in Section 4.4, re-expands all the nodes in the search branch, after an action has been pushed up. We can see that running *AltAlt*^{p} with *PushUp* and fattening procedure is better than just the latter. In Figure 21(b) we can see that although the *PushUp* procedure does not add much overhead, the aggressive version of *PushUp* does get quite expensive. We also notice that only around 20 problems are solved within time limits with aggressive *PushUp*. We can

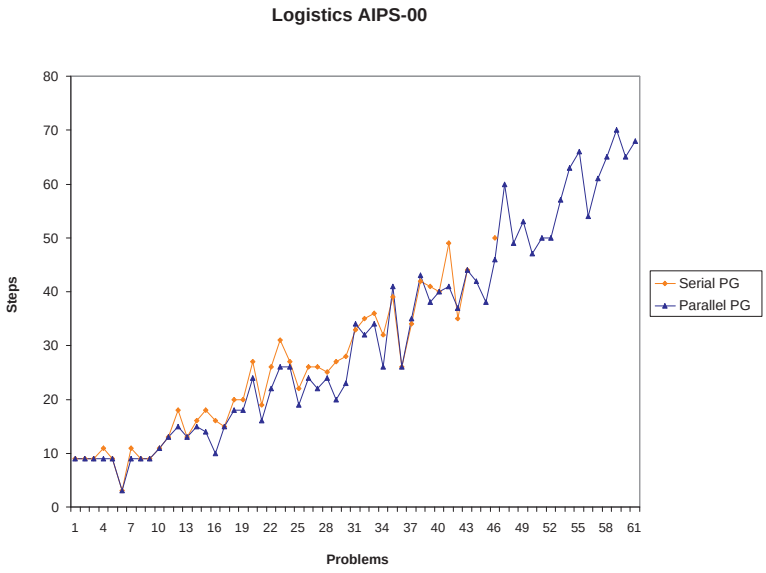


(a)

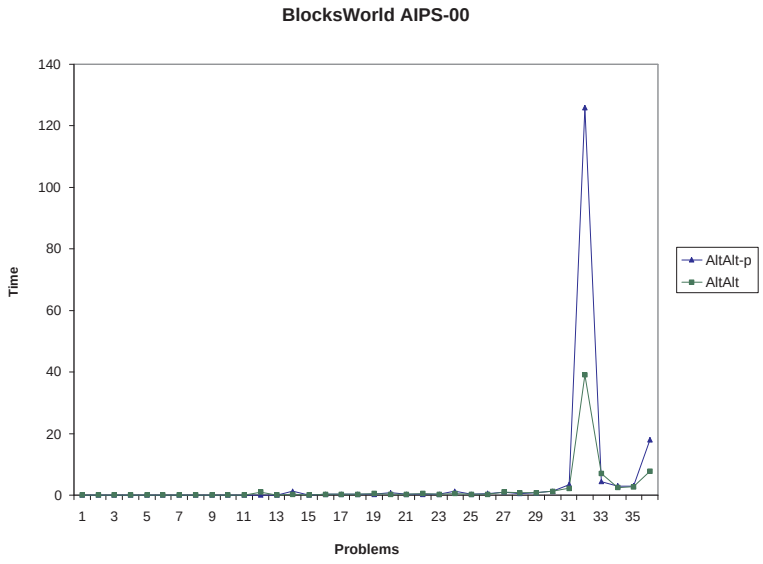


(b)

Figure 21. Analyzing the effect of the *PushUp* procedure on the Logistics domain



(a) Utility of using Parallel Planning Graphs



(b) Solving a Serial domain

Figure 22. Plots showing the utility of using parallel planning graphs in computing the heuristics, and characterizing the overhead incurred by $AltAlt^p$ in serial domains.

conclude then that the *PushUp* procedure, used to offset the greediness of the algorithm, achieves its purpose.

Utility of basing heuristics on Parallel Planning Graphs: We can see in Figure 22(a) that using parallel planning graph as the basis for deriving heuristic estimates in $AltAlt^p$ is a winning idea. The serial planning graph overestimates the heuristic values in terms of steps, producing somewhat longer parallel solutions. The fact that the version using serial planning graph runs out of time in many problems also demonstrates that the running times are also improved by the use of parallel planning graphs.

Comparison to *AltAlt*: One final concern would be how much of an extra computational hit is taken by the $AltAlt^p$ algorithm in serial domains (e.g. Blocks World [Bacchus, 2001]). We expect it to be negligible since $O = \emptyset$. To confirm our intuitions, we ran $AltAlt^p$ on a set of problems from the sequential Blocks-world domain from [Bacchus, 2001]. We see from the plot 22(b) that the time performance between *AltAlt* and $AltAlt^p$ are equivalent for almost all of the problems.[4]

[4]For a deeper analysis and more experiments see [Sanchez and Kambhampati, 2003a].

CHAPTER 5

Planning Graph Based Heuristics for Partial Satisfaction (Over-subscription) Planning

Most planners handle goals of attainment, where the objective is to find a sequence of actions that transforms a given initial state I to some goal state G, where $G = g_1 \wedge g_2 \wedge ... \wedge g_n$ is a conjunctive list of goal fluents. Plan success for these planning problems is measured in terms of whether or not all the conjuncts in G are achieved. However, in many real world planning scenarios, the agent may only be able to partially satisfy G, because of subgoal interactions, or lacking of resources and time. Effective handling of partial satisfaction planning (PSP) problems poses several new challenges, including the problem of designing efficient goal selection heuristics, and an added emphasis on the need to differentiate between feasible and optimal plans based on new models for handling plan quality (in terms of action costs and goal utilities). In this chapter, We provide a first systematic analysis of PSP problems. We will start by distinguishing several classes of PSP problems, but focus on one of the most general PSP problems, called PSP Net Benefit. In this problem, each goal conjunct has a fixed utility attached to it, and each ground action

has a fixed execution cost associated with it. The objective is to find a plan with the best "net benefit" (i.e., cumulative utility minus cumulative cost).

Despite the ubiquity of PSP problems, surprisingly little attention has been paid to the development of effective approaches for solving them in the planning community. Earlier work by the PYRRHUS system [Williamson and Hanks, 1994] allows for partial satisfaction in planning problems with goal utilities. However, unlike the PSP problems discussed in this Dissertation, PYRRHUS requires all goals to be achieved; partial satisfaction is interpreted by using a non-increasing utility function on each of the goals. Many NASA planning problems have been identified as partial satisfaction problems [Smith, 2003]. Some preliminary work by Smith (2004) proposed a planner for over-subscribed planning problems, in which an abstract planning problem (i.e., the orienteering graph) is built to select the subset of goals and orderings to achieve them. Smith (2004) speculated that the heuristic distance estimates derived from a planning graph data structure are not particularly suitable for PSP problems given that they make the assumption that goals are independent, without solving the problem of interactions among them.

In this chapter, We show that in fact although planning graph estimates for PSP problems are not very accurate in the presence of complex goal interactions, they can also be extended to overcome such problems. In particular, We extend the reachability analysis provided by planning graphs to compute cost-sensitive heuristics augmented with mutex analysis to overcome over-subscribed planning problems. Our approach named *AltWlt* [Sanchez and Kambhampati, 2005], involves a sophisticated multiple goal set selection process augmented with mutex analysis in order to solve complex PSP problems. The goal set selection process considers multiple combinations of goals and assigns penalty costs

based on mutex analysis when interactions are found. Once a subset of goal conjuncts is selected, they are solved by a regression search planner with cost-sensitive planning graph heuristics.

The rest of this chapter is organized as follows. First, We start by describing a spectrum of PSP problems, and focus on the PSP NET BENEFIT problem, where actions have execution costs and goals have utilities. After that, We will review *AltAlt*^{ps} [van den Briel *et al.*, 2004b; 2004a], our initial approach to PSP, which forms the basis for *AltWlt*, emphasizing its goals set selection algorithm and cost-sensitive reachability heuristics. In the next part of this chapter, We will introduce *AltWlt*, pointing out the limitations of *AltAlt*^{ps} with some clarifying examples, and the extensions to overcome them. After that, We will present a set of complex PSP problems and an empirical study on them that compares the effectiveness of *AltWlt* with respect to its predecessor, and another experimental PSP based planners. Finally, We conclude this chapter with some discussion on the current state of the art alternative approaches, including among these *Sapa*^{ps} [Do and Kambhampati, 2004], another planning-graph based heuristic planner, and the optimal MDP and IP formulation to PSP NET BENEFIT to assess the quality of the solutions returned by *AltWlt*.

5.1. Problem Definition and Complexity

The following notation will be used: F is a finite set of fluents and A is a finite set of actions, where each action consists of a list of preconditions and a list of add and delete effects. $I \subseteq F$ is the set of fluents describing the initial state and $G \subseteq F$ is the set of goal conjuncts. Hence we define a planning problem as a tuple $P = (F, A, I, G)$. Figure 23

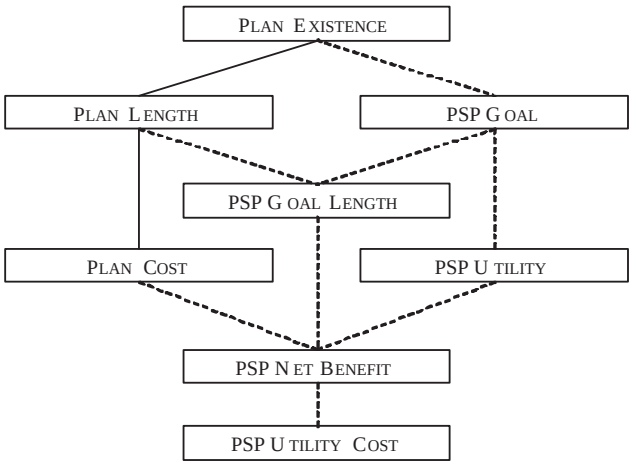


Figure 23. Hierarchical overview of several types of complete and partial satisfaction planning problems.

gives a taxonomic overview of several types of complete and partial satisfaction planning problems. The problem of PSP NET BENEFIT is a combination of the problem of finding minimum cost plans (PLAN COST) and the problem of finding plans with maximum utility (PSP UTILITY), as a result is one of the most general PSP problems.[1] In the following, We formally define the problem of finding a plan with maximum net benefit:

Definition 7 (PSP Net Benefit:)**.** *Given a planning problem* $P = (F, A, I, G)$ *and, for each action a "cost"* $\mathcal{C}_a \geq 0$ *and, for each goal specification* $f \in G$ *a "utility"* $\mathcal{U}_f \geq 0$*, and a positive number* k*. Is there a finite sequence of actions* $\Delta = \langle a_1, ..., a_n \rangle$ *that starting from* I *leads to a state* S *that has net benefit* $\sum_{f \in (S \cap G)} \mathcal{U}_f$ - $\sum_{a \in \Delta} \mathcal{C}_a \geq k$*?*

Given that PLAN EXISTENCE and PSP NET BENEFIT are PSPACE-hard problems [van den Briel *et al.*, 2004b; 2004a], it should be clear that the other problems given in Figure 23 also fall in this complexity class. PSP NET BENEFIT does, however, foreground

[1] For a more comprehensive study on the complexity and taxonomy of PSP problems see [van den Briel *et al.*, 2004b; 2004a].

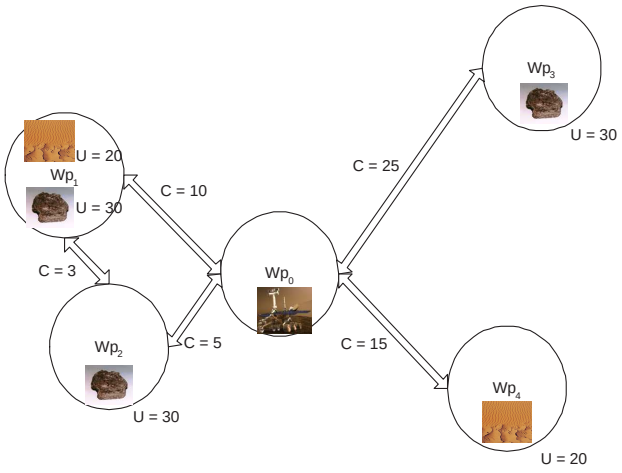


Figure 24. Rover domain problem

the need to handle plan quality issues. To clarify the PSP NET BENEFIT problem, we introduce the following example:

Example 5.1

Figure 24 illustrates again a small example from the rover domain [Long and Fox, 2003] to motivate the need for partial satisfaction. [2] This time, a rover that has landed on Mars needs to collect scientific data from some rock and soil samples. Some waypoints have been identified on the surface. Each waypoint has scientific samples. For example, $waypoint_3$ has a rock sample, while $waypoint_4$ has a soil sample. The rover needs to travel to a corresponding waypoint to collect its samples. Each travel action has a cost associated to it. For example, the cost of traveling from $waypoint_0$ to $waypoint_1$ is given by $\mathcal{C}_{travel_{0,1}} = 10$. In addition to the $travel_{x,y}$ actions, we have two more actions *sample* and *comm* to collect and communicate the data respectively to the lander. To simplify our problem, these actions have uniform costs independent of the locations where they take

[2] This time the notation of the problem is more descriptive to better understand it.

place. These costs are specified by $\mathcal{C}_{sample_{data,x}} = 5$ and $\mathcal{C}_{comm_{data,x,y}} = 4$. Each sample (or subgoal) has a utility attached to it. We have a utility of $\mathcal{U}_{rock_3} = 30$ for the rock sample at $waypoint_3$, and a utility $\mathcal{U}_{soil_4} = 20$ for a soil sample at $waypoint_4$. The goal of the rover is to find a travel plan that achieves the best cost-utility tradeoff for collecting the samples. In this example, the best plan is $P = \{travel_{0,2}$, $sample_{rock_2,2}$, $comm_{rock_2,2,0}$, $travel_{2,1}$, $sample_{rock_1,1}$, $comm_{rock_1,1,0}$, $sample_{soil_1,1}$, $comm_{soil_1,1,0}\}$ which achieves the goals $rock_2, rock_1$ and $soil_1$, and ignores the rest of the samples at $waypoint_3$ and $waypoint_4$ giving the net benefit 45.

As mentioned before, current planning frameworks do not address the issue of partial satisfaction of goals, because they strictly try to satisfy a conjunction of them. However, this all-or-nothing success criterion does not seem to be very realistic. There are problems where satisfying at least some of the goals is better than not satisfying any of them at all. The problem is of course to choose the goals that will be satisfied with respect to an optimization criteria. PSP problems could be solved optimally using Integer Programming techniques [Haddawy and Hanks, 1993]. Unfortunately, current IP planning approaches do not scale up well to bigger sized problems [Kautz and Walser, 1999; Vossen *et al.*, 1999; van den Briel *et al.*, 2004b]. In the following section, we introduce $AltAlt^{ps}$, our preliminary heuristic approach to Partial Satisfaction (Over-subscription) Planning. First, We describe how the cost information is propagated in a planning graph data structure, and then how this information is used to select the set of goals upfront and guide the search of the planner.

5.2. Background: *AltAlt^{ps}* Cost-based Heuristic Search and Goal Selection

In this section, We introduce *AltAlt^{ps}* because it forms the basis for *AltWlt*, the proposed algorithm that handles complex goal interactions. *AltAlt^{ps}* is a heuristic regression planner that can be seen as a variant of *AltAlt* [Sanchez *et al.*, 2000; Nguyen *et al.*, 2002] equipped with cost sensitive heuristics. An obvious, if naive, way of solving the PSP NET BENEFIT problem with such a planner is to consider all plans for the 2^n subsets of an n-*goal* problem, and see which of them will wind up leading to the plan with the highest net benefit. Since this is infeasible, *AltAlt^{ps}* uses a greedy approach to pick a goal subset up front. The greediness of the approach is offset by considering the net benefit of covering a goal not in isolation, but in the context of the potential (relaxed) plan for handling the already selected goals. Once a subset of goal conjuncts is selected, *AltAlt^{ps}* finds a plan that achieves such subset using its regression search engine augmented with cost sensitive heuristics. This description can be seen in the overall architecture of *AltAlt^{ps}* in Figure 25.

Given that the quality of the plans for PSP problems depends on both the utility of the goals achieved and the cost to achieve them, *AltAlt^{ps}* needs heuristic guidance that is sensitive to both action cost and goal utility. Because only the execution costs of the actions and the achievement cost of propositions in the initial state (zero cost) are known, we need to do *cost-propagation* from the initial state through actions to estimate the cost to achieve other propositions, especially the top level goals. We can see in Figure 25 that *AltAlt^{ps}* is using the planning graph structure to compute this cost information. This information over the planning graph is the basis for heuristic estimation in *AltAlt^{ps}*, and is also used to estimate the most beneficial subset of goals upfront and guide the search in the planner.

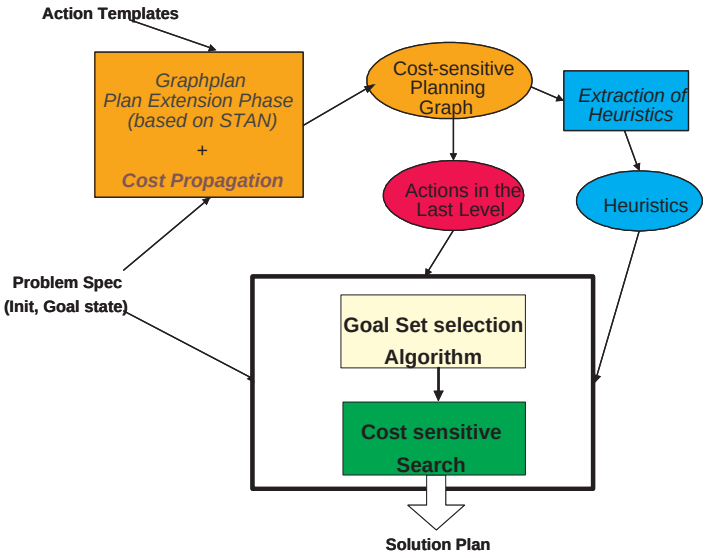


Figure 25. *AltAlt*^{ps} architecture

The cost sensitive heuristics, as well as the goal set selection algorithm are described in more detail in the next sections.

5.2.1. Propagating Cost as the Basis for Computing Heuristics. Following [Do and Kambhampati, 2003], cost functions are used to capture the way cost of achievement changes as the graph gets expanded. In the following, we briefly review the procedure.

The purpose of the cost-propagation process is to build the cost functions $C(f, l_f)$ and $C(a, l_a)$ that estimate the cheapest cost to achieve fluent f at level l_f of the planning graph, and the cost to execute action a at level l_a. At the beginning ($l = 0$), let S_{init} be the initial state and $\mathcal{C}_a$ be the cost of action a then[3]: $C(f, 0) = 0$ if $f \in S_{init}$, $C(f, 0) = \infty$ otherwise; $\forall a \in A : C(a, 0) = \infty$. The propagation rules are as follows:

[3] $\mathcal{C}_a$ and $C(a, l)$ are different. If $a = Travel_{0,1}$ then $\mathcal{C}_a$ is the travel cost and $C(a, l)$ is the cost to achieve preconditions of a at level l, which is the cost incurred to be at $waypoint_0$ at l.

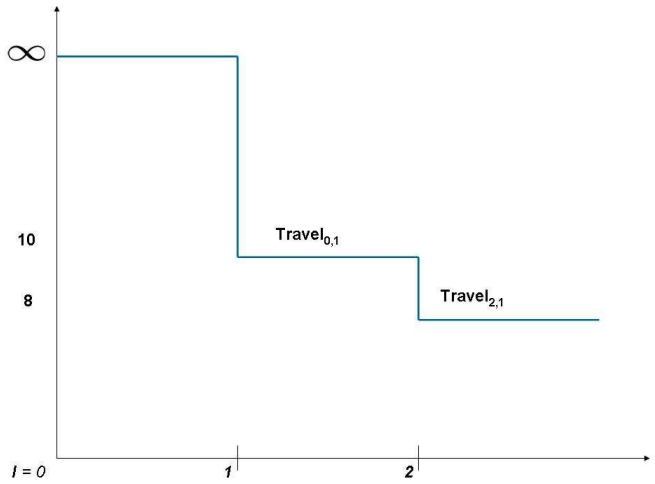


Figure 26. Cost function of $at(waypoint_1)$

- $C(f,l) = min\{C(a,l) + \mathcal{C}_a) : f \in Eff(a)\}$
- Max-prop: $C(a,l) = max\{C(f,l) : \ f \in Prec(a)\}$
- Sum-prop: $C(a,l) = \Sigma\{C(f,l) : \ f \in Prec(a)\}$

The max-propagation rule will lead to an admissible heuristic, while the sum-propagation rule does not. Assume that we want to reach $waypoint_1$ in our rover example. We can reach it directly from $waypoint_0$ within a unit of time, or we can travel through $waypoint_2$ and reach it within two steps. Figure 26 shows the cost function for proposition $p_1 = At(waypoint_1)$, which indicates that the earliest level to achieve p_1 is at $l = 1$ with the lowest cost of 10 (route: $waypoint_0 \rightarrow waypoint_1$). The lowest cost to achieve p_1 reduces to 8 at $l = 2$ (route: $waypoint_0 \rightarrow waypoint_2 \rightarrow waypoint_1$) for the leveled planning graph.

There are many ways to terminate the cost-propagation process [Do and Kambhampati, 2003]: We can stop when all the goals are achievable, when the cost of all the goals are stabilized (i.e. guaranteed not to decrease anymore), or lookahead several steps after

the goals are achieved. For classical planning, we can also stop propagating cost when the graph *levels-off* [Nguyen *et al.*, 2002].

5.2.2. Cost-sensitive Heuristics. Before describing the goal set selection process of $AltAlt^{ps}$, We introduce in this section cost-based heuristics that will help us to guide the search of the planner once the goals have been selected. Notice that from the cost propagation procedure described in the last section, we could easily derive the first heuristic for our cost-based planning framework, computed under the assumption that the propositions constituting a state are strictly independent. Such a heuristic is described as follows:

Sum Cost Heuristic 1. $h_{sumC}(S) = \sum_{p \in S} C(p, l)$[4]

The h_{sumC} heuristic suffers from the same problems than the h_{sum} heuristic introduced by Bonet *et al.*,1997. It will tend to overestimate the cost of a set of propositions. To make the heuristic admissible we could replace the *sum* function with the maximum of the individual costs of the propositions composing the state. This leads us to our next heuristic:

Max Cost Heuristic 2. $h_{maxC}(S) = \max_{p \in S} C(p, l)$

This heuristic also directly resembles the h_{max} heuristic from [Bonet *et al.*, 1997; Nguyen *et al.*, 2002], but in terms of cost. We could try to combine the *differential* power of h_{sumC} and h_{maxC} to get a more effective heuristic for a wider range of problems [Nguyen *et al.*, 2002].

[4] *Where* $l = levels - off$.

Combo Cost Heuristic 3. $h_{comboC}(S) = h_{sumC}(S) + h_{maxC}(S)$

We could improve further the solution quality of our heuristics if we start taking into account the positive interactions among subgoals while still ignoring the negative ones. We could adapt the idea of the relaxed plan heuristic from [Hoffmann and Nebel, 2001; Nguyen and Kambhampati, 2001; Nguyen *et al.*, 2002] into our framework. So, $AltAlt^{ps}$ uses variations of the relaxed plan extraction process guided by the cost-functions to estimate heuristic values $h(S)$ [Do and Kambhampati, 2003]. The basic idea is to compute the cost of the relaxed plans in terms of the costs of the actions comprising them, and use such costs as heuristic estimates. The general relaxed plan extraction process for $AltAlt^{ps}$ works as follows:

1. Start from the goal set G containing the top level goals, remove a goal g from G and select a lowest cost action a_g (indicated by $C(g, l)$) to support g

2. Regress G over action a_g, setting $G = G \cup Prec(a_g) \backslash Eff(a_g)$

The process above continues recursively until each proposition $q \in G$ is also in the initial state I. This regression accounts for the positive interactions in the state G given that by subtracting the effects of a_g, any other proposition that is co-achieved when g is being supported is not counted in the cost computation. The relaxed plan extraction procedure indirectly extracts a sequence of actions R_P, which would have achieved the set G from the initial state I if there were no negative interactions. The summation of the costs of the actions $a_g \in R_P$ can be used to estimate the cost to achieve all goals in G, in summary we have:

Relax Cost Heuristic 4. $h_{relaxC}(S) = \sum_{a \in R_p} \mathcal{C}_a$

5.3. *AltAlt*^{ps} Goal Set Selection Algorithm

The main idea of the goal set selection procedure in *AltAlt*^{ps} is to incrementally construct a new partial goal set G' from the top level goals G such that the goals considered for inclusion increase the final net benefit, using the goals utilities and costs of achievement. The process is complicated by the fact that the net benefit offered by a goal g depends on what other goals have already been selected. Specifically, while the utility of a goal g remains constant, the expected cost of achieving it will depend upon the other selected goals (and the actions that will anyway be needed to support them). To estimate the "residual cost" of a goal g in the context of a set of already selected goals G', we compute a relaxed plan R_P for supporting $G' + g$, which is biased to (re)use the actions in the relaxed plan R'_P for supporting G'.

Figure 27 gives a description of the goal set selection algorithm. The first block of instructions before the loop initializes our goal subset G',[5] and finds an initial relaxed plan R^*_P for it using the procedure *extractRelaxedPlan(G',∅)*. Notice that two arguments are passed to the function. The first one is the current partial goal set from where the relaxed plan will be computed. The second parameter is the current relaxed plan that will be used as a guidance for computing the new relaxed plan. The idea is that we want to bias the computation of the new relaxed plan to re-use the actions in the relaxed plan from the previous iteration. Having found the initial subset G' and its relaxed plan R^*_P, we

[5] $getBestBenefitialGoal(G)$ returns the subgoal with the best benefit, $\mathcal{U}_g - C(g, l)$ tradeoff

```
Procedure GoalSetSelection(G)
    g ← getBestBenefitialGoal(G);
    if(g = NULL)
        return Failure;
    G' ← {g}; G ← G \ g;
    R*_P ← extractRelaxedPlan(G', ∅)
    B*_MAX ← getUtil(G') − getCost(R*_P);
    B_MAX ← B*_MAX
    while(B_MAX > 0 ∧ G ≠ ∅)
        for(g ∈ G \ G')
            G_P ← G' ∪ g;
            R_P ← ExtractRelaxedPlan(G_P, R*_P)
            B_g ← getUtil(G_P) − getCost(R_P);
            if(B_g > B*_MAX)
                g* ← g; B*_MAX ← B_g; R*_g ← R_P;
            else
                B_MAX ← B_g − B*_MAX
        end for
        if(g* ≠ NULL)
            G' ← G' ∪ g*; G ← G \ g*; B_MAX ← B*_MAX;
    end while
    return G';
End GoalSetSelection;
```

Figure 27. Goal set selection algorithm.

compute the current best net benefit B^*_{MAX} by subtracting the costs of the actions in the relaxed plan R^*_P from the total utility of the goals in G'. B^*_{MAX} will work as a threshold for our iterative procedure. In other words, we would continue adding subgoals $g \in G$ to G' only if the overall net benefit B^*_{MAX} increases. We consider one subgoal at a time, always computing the benefit added by the subgoal in terms of the cost of its relaxed plan R_P and goal utility B_g. We then pick the subgoal g that maximizes the net benefit, updating the necessary values for the next iteration. This iterative procedure stops as soon as the net benefit does not increase, or when there are no more subgoals to add, returning the new goal subset G'.

In our running example, the original subgoals are $\{g_1 = soil_1, \ g_2 = rock_1, \ g_3 = rock_2, \ g_4 = rock_3, \ g_5 = soil_4\}$, with final costs $C(g,t) = \{17, 17, 14, 34, 24\}$ and utilities vectors $\mathcal{U} = \{20, 30, 30, 30, 20\}$ respectively, where $t = leveloff$ in the planning graph. Following our algorithm, our starting goal g would be g_3 because it returns the biggest benefit (e.g. 30 - 14). Then, G' is set to g_3, and its initial relaxed plan R^*_P is computed. Assume that the initial relaxed plan found is $R^*_P = \{travel_{0,2}, \ sample_{rock_2}, \ comm_{rock_2,2,0}\}$. We proceed to compute the best net benefit using R^*_P , which in our example would be $B^*_{MAX} = 30 - (5 + 5 + 4) = 16$. Having found our initial values, we continue iterating on the remaining goals $G = \{g_1, g_2, g_4, g_5\}$. On the first iteration we compute four different set of values, they are: (i) $G_{P_1} = \{g_3 \cup g_1\}$, $R_{P_1} = \{travel_{2,1}, \ sample_{soil_1}, \ comm_{soil_1,2,0}, \ travel_{0,2}, \ sample_{rock_2}, \ comm_{rock_2,2,0}\}$, and $B_{g_{p_1}} = 24$; (ii) $G_{P_2} = \{g_3 \cup g_2\}$, $R_{P_2} = \{travel_{2,1}, \ sample_{rock_1}, \ comm_{rock_1,2,0}, \ travel_{0,2}, \ sample_{rock_2}, \ comm_{rock_2,2,0}\}$, and $B_{g_{p_2}} = 34$; (iii) $G_{P_3} = \{g_3 \cup g_4\}$, $R_{P_3} = \{travel_{0,3}, \ sample_{rock_3}, \ comm_{rock_3,2,0}, \ travel_{0,2}, \ sample_{rock_2}, \ comm_{rock_2,2,0}\}$, and $B_{g_{p_3}} = 12$, and (iv) $G_{P_4} = \{g_3 \cup g_5\}$, $R_{P_4} = \{travel_{0,4},$

$sample_{soil_4}$, $comm_{soil_4,2,0}$, $travel_{0,2}$, $sample_{rock_2}$, $comm_{rock_2,2,0}\}$ with $B_{g_{p_4}} = 12$. Notice then that our net benefit B^*_{MAX} could be improved most if we consider goal g_2. So, we update $G' = g_3 \cup g_2$, $R^*_P = R_{P_2}$, and $B^*_{MAX} = 34$. The procedure keeps iterating until only g_4 and g_5 remain, which decrease the final net benefit. The procedure returns then $G' = \{g_1, g_2, g_3\}$ as our goal set, which in fact it is the optimal goal set. In this example, there is also a plan that achieves the five goals with a positive benefit, but it is not as good as the plan that achieves the selected G'.

5.4. *AltWlt*: Extending *AltAlt*^{ps} to Handle Complex Goal Scenarios

The advantage of *AltAlt*^{ps} for solving PSP problems is that after committing to a subset of goals, the overall problem is simplified to the planning problem of finding the least cost plan to achieve the goal set selected, avoiding the exponential search on 2^n goal subsets. However, the goal set selection algorithm of *AltAlt*^{ps} is greedy, and as a result it is not immune from selecting a bad subset. The main problem with the algorithm is that it does not consider goal interactions. Because of this limitation the algorithm may:

- return a wrong initial subgoal affecting the whole selection process, and
- select a set of subgoals that may not even be achievable due to negative interactions among them.

The first problem corresponds to the selection of the initial subgoal g from where the final goal set will be computed, which is one of the critical decisions of the algorithm. Currently, the algorithm selects only the subgoal g with the highest positive net benefit.

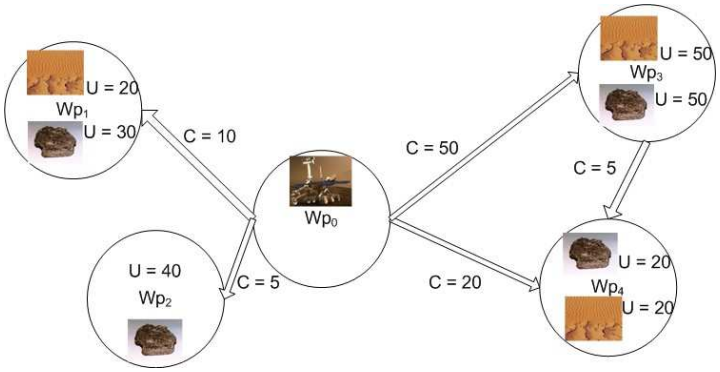


Figure 28. Modified rover example with goal interactions

Although, this first assumption seems to be reasonable, there may be situations in which starting with the most promising goal may not be the best option. Specifically, when a large action execution cost is required upfront to support a subset of the top level goals, in which each isolated goal component in the subset would have a very low benefit estimate (even negative), precluding the algorithm for considering them initially, but in which the conjunction of them could return a better quality solution. The problem is that we are considering each goal individually in the beginning, without looking ahead into possible combinations of goals in the heuristic computation.

Example 5.4

Consider again the modified Rover example from Figure 28. This time, we have added extra goals, and different cost-utility metrics to our problem. Notice also that the traversal of the paths has changed. For example, we can travel from $waypoint_0$ to $waypoint_1$, but we can not do the reverse. Our top-level goals are $\{g_1 = soil_1,\ g_2 = rock_1,\ g_3 = rock_2,\ g_4 = rock_3,\ g_5 = soil_3,\ g_6 = rock_4,\ g_7 = soil_4\}$, with final costs $C(g,t) = \{19, 19, 14, 59, 59, 29, 29\}$ and utilities $\mathcal{U} = \{20, 30, 40, 50, 50, 20, 20\}$ respectively. Following this example, the goal set selection algorithm would choose goal g_3 as its initial subgoal because it returns the

highest net benefit (e.g. 40 - 14). Notice this time that considering the most promising subgoal is not the best option. Once the rover reaches $waypoint_2$, it can not achieve any other subgoal. In fact, there is a plan P for this problem with a bigger net benefit that involves going to $waypoint_3$, and then to $waypoint_4$ collecting their samples. Our current goal selection algorithm can not detect P because it ignores the samples on such waypoints given that they do not look individually better than g_3 (e.g. g_4 has a benefit of $50 - 59$). This problem arises because the heuristic estimates derived from our planning graph cost propagation phase assume that the goals are independent, in other words, they may not provide enough information if we want to achieve several consecutive goals.

The second problem about negative interactions among goals is also exhibited in the last example. We already mentioned that if we choose g_3 we can not select any other goal. However, our goal set selection algorithm would also select g_1 and g_2 given that the residual cost returned by the relaxed plan heuristic is lower than the benefit added because it ignores the negative interactions among goals. So, our final goal set would be $G = \{g_3, g_2, g_1\}$, which is not even achievable. Clearly, we need to identify such goal interactions and add some cost metric when they exist. □

We extended the goal selection algorithm in *AltWlt* to overcome these problems. Specifically, this work considers multiple groups of subgoals, in which each subgoal from the top level goal set is forced to be true in at least one of the groups, and also considers adding penalty costs based on mutex analysis to account for complex interactions among goals to overcome the limitations of the relaxed plan heuristic. Although these problems could be solved independently, they can be easily combined and solved together. These additions are discussed in the next sections.

Procedure $MGS(G)$
$B^*_{MAX} \leftarrow -\infty$, $G^* \leftarrow \emptyset$
for $(g_i \in G)$
$GL_i \leftarrow nonStaticMutex(g_i, G \setminus g_i)$
$Rp_i \leftarrow extractGreedyRelaxedPlan(g_i, \emptyset)$
$G'_i \leftarrow greedyGoalSetSelection(g_i, GL_i, Rp_i)$
$NB_i \leftarrow getUtil(G'_i) - getCost(Rp_i)$
if$(NB_i > B^*_{MAX})$
$B^*_{MAX} \leftarrow NB_i$, $G^* \leftarrow G'_i$
end for
return G^*;
End MGS;

Figure 29. Multiple goal set selection algorithm

5.4.1. Goal Set Selection with Multiple Goal Groups. The general idea behind the goal set selection with multiple groups procedure is to consider each goal g_i from the top level goal set G as a feasible starting goal, such that we can be able to find what the benefit would be if such goal g_i were to be part of our final goal set selected. The idea is to consider more aggressively multiple combinations of goals in the selection process. Although, we relax the assumption of having a positive net benefit for our starting goals, the approach is still greedy. It modifies the relaxed plan extraction procedure to bias not only towards those actions found in the relaxed plan of the previous iteration, but also towards those facts that are reflected in the history of partial states of the previous relaxed plan computation to account for more interactions. The algorithm will stop computing a new goal set as soon as the benefit returned decreases. The new algorithm also uses mutex analysis to avoid computing non-achievable goal groups. The output of the algorithm is the goal group that maximizes our net benefit. A more detailed description of the algorithm is shown in Figure 29, and is discussed below.

Given the set of top level goals G, the algorithm considers each goal $g_i \in G$ and finds a corresponding goal subset G'_i with positive net benefit. To get such subset, the algorithm uses a modified greedy version of our original *GoalSetSelection* function (from Figure 27), in which the goal g_i has been set as the initial goal for G'_i, and the initial relaxed plan Rp_i for supporting g_i is passed to the function. Furthermore, the procedure only considers those top-level goals left $GL_i \subseteq G$ which are not pair-wise static mutex with g_i. The set GL_i is obtained using the procedure *nonStaticMutex* in the algorithm. By selecting only the non-static mutex goals, we partially solve the problem of negative interactions, and reduce the running time of the algorithm. However, we still need to do additional mutex analysis to overcome complex goal interactions (e.g. dynamic mutexes); and we shall get back to this below. At each iteration, the algorithm will output a selected goal set G'_i given g_i, and a relaxed plan Rp_i supporting it.

As mentioned in the beginning of this section, the modified *extractGreedyRelaxedPlan* procedure takes into account the relaxed plan from the previous iteration (e.g. Rp_i^*) as well as its partial execution history to compute the new relaxed plan Rp_i for the current subgoal g_i. The idea is to adjust the aggregated cost of the actions $C(a_i, l)$ supporting g_i, to order them for inclusion in Rp_i, when their preconditions have been accomplished by the relaxed plan Rp_i^* from the previous iteration. Remember that $C(a_i, l)$ has been computed using our cost propagation rules, we decrease this cost when $Prec(a_i) \cap_{\exists a_k \in Rp_i^*} Eff(a_k) \neq \emptyset$ is satisfied. In other words, if our previous relaxed plan Rp_i^* supports already some of the preconditions of a_i it better be the case that such preconditions are not being over-counted in the aggregated cost of the action a_i. This greedy modification of our relaxed plan extraction procedure biases even more to our

previous relaxed plans, ordering differently the actions that will be used to support our current subgoal g_i. The idea is to try to adjust the heuristic positively to overcome the independence assumption among subgoals.

For example, on Figure 28, assume that our previous relaxed plan Rp_i^* has achieved the subgoals at $waypoint_3$, and we want to achieve subgoal $g_i = soil_4$. In order to collect the sample, we need to be at $waypoint_4$, the cheapest action in terms of its aggregated cost that supports that condition is $a = travel_{0,4}$ with cost of 20 which precludes g_i for being considered (no benefit added). However, notice that there is another action $b = travel_{3,4}$ with original aggregated cost of 50 (due to its precondition), whose cost gets modified by our new relaxed plan extraction procedure since its precondition (at $waypoint_3$) is being supported indirectly by our previous Rp_i^*. By considering action b, the residual cost for supporting $soil_4$ lowers to 5, and as a result it can be considered for inclusion.

Finally, the algorithm will output the goal set G^* that maximizes the net benefit B^*_{MAX} among all the different goal partitions G'_i. Following Example 2 from Figure 28, we would consider 7 goal groups having the following partitions: $g_1 = soil_1$ & $GL_1 = \{g_2\}$, $g_2 = rock_1$ & $GL_2 = \{g_1\}$, $g_3 = rock_2$ & $GL_3 = \emptyset$, $g_4 = rock_3$ & $GL_4 = \{g_5, g_6, g_7\}$, $g_5 = soil_3$ & $GL_5 = \{g_4, g_6, g_7\}$, $g_6 = rock_4$ & $GL_6 = \{g_7\}$, $g_7 = soil_4$ & $GL_7 = \{g_6\}$. The final goal set returned by the algorithm in this example is $G^* = \{g_4, g_5, g_6, g_7\}$, which corresponds to the fourth partition G'_4, with maximum benefit of 49. Running the original algorithm (from Figure 27) in this example would select goal group $G'_3 = \{g_3\}$ with final benefit of 26.

Even though our algorithm may look expensive since it is looking at different goal combinations on multiple groups, it is still a greedy approximation of the full 2^n combina-

tions of an optimal approach. The reduction comes from setting up the initial subgoals at each goal group at each iteration. The worst case scenario of our algorithm would involve to consider problems with no interactions and high goal utility values, in which the whole set of remaining subgoals would need to be considered at each group. Given n top level goals leading to n goal groups, the worst case running time scenario of our approach would be in terms of $n * \sum_{i=1}^{n-1} i$, which is much better than the factor 2^n.

Notice that our multiple goal set selection process shares some similarities to the *Partition-k* heuristic introduced in Section 3.5. The *Partition-k* heuristic tries to build subsets of interacting goals, and then adds the amount of interactions found for each particular subset built. The idea is to measure how costly a state is in terms of its subsets interactions. Although, our *MGS* algorithm also takes into account subgoals interactions, it indirectly behaves in an opposite direction to the *Partition-k* heuristic. The basic idea of *MGS* is to keep adding subgoals to each goal subset if they increase the net benefit returned. Usually, the subgoals added are those that are less costlier, which most of the times are also less interacting with the current subset of goals. In other words, while *MGS* tries to measure how beneficial a subset is, *Partition-k* considers how costly a subset could be.

5.4.2. Penalty Costs Through Mutex Analysis. Although the *MGS* algorithm considers static mutexes, it still misses negative interactions among goals that could affect the goal selection process. This is mainly due to the optimistic reachability analysis provided by the planning graph. Consider again Example 5.4, and notice that goals $g_5 = soil_3$ and $g_7 = soil_4$ are not statically interfering, and they require a minimum of three steps (actions) to become true in the planning graph (e.g. *travel* - *sample* - *comm*). However, at

level 3 of the planning graph these goals are mutex, implying that there are some negative interactions among them. Having found such interactions, we could assign a *penalty cost* P_C to our residual cost estimate for ignoring them.

5.4.2.1. *Penalty Costs Through Subgoal Interactions.* A first approach for assigning such a penalty cost P_C, which we call $NEGFM$,[6] follows the work from [Nguyen *et al.*, 2002] considering the binary interaction degree δ among a pair of propositions. The idea is that every time a new subgoal g gets considered for inclusion in our goal set G', we compute δ among g and every other subgoal $g' \in G'$. At the end, we output the pair $[g, g']$ with highest interaction degree δ if any. Recalling from Chapter 3, δ gets computed using the following equation:

$$\delta(p_1, p_2) = lev(p_1 \wedge p_2) - max\{lev(p_1), lev(p_2)\} \tag{5.1}$$

Where $lev(S)$ corresponds to the set level heuristic that specifies the earliest level in the planning graph in which the propositions in the set S appear and are not mutex to each other [Nguyen *et al.*, 2002]. Obviously, if not such level exists then $lev(S) = \infty$, which is the case for static mutex propositions.

The binary degree of interaction δ provided a clean way for assigning a penalty cost to a pair of propositions in the context of heuristics based on number of actions, given that δ was representing the number of extra steps (actions) required to make such pair of propositions mutex free in the planning graph. Following our current example, $lev(g_5 \wedge g_7) = 5$ (due to dummy actions), as a result $\delta(g_5, g_7) = 2$ represents the cost for

[6] Negative Factor: Max

ignoring the interactions. However, in the context of our PSP problem, where actions have real execution costs and propositions have costs of achievement attached to them, it is not clear how to compute a penalty cost when negative interactions are found.

Having found the pair with highest $\delta(g, g')_{g' \in G'}$ value, our first solution $NEGFM$ considers the maximum cost among both subgoals in the final level l_{off} of the planning graph as the penalty cost P_C for ignoring such interaction. This is defined as:

$$P_C(g, G')_{NEGFM} = max \left\{ \begin{array}{c} (C(g, l_{off}), C(g', l_{off})) \\ \\ : g' \in G' \wedge max(\delta(g, g')) \end{array} \right\} \tag{5.2}$$

$NEGFM$ is greedy in the sense that it only considers the pair of interacting goals with maximum δ value. It is also greedy in considering only the maximum cost among the subgoals in the interacting pair as the minimum amount of extra cost needed to overcome the interactions generated by the subgoal g being evaluated. Although $NEGFM$ is easy to compute, it is not very informative affecting the quality of the solutions returned. The main reason for this is that we have already considered partially the cost of achieving g when its relaxed plan is computed, and we are in some sense blindly over-counting the cost if $C(g, l_{off})$ gets selected as the penalty P_C. Despite these clear problems, $NEGFM$ is able to improve in problems with complex interactions over our original algorithm.

5.4.2.2. *Penalty costs through action interactions.* A better idea for computing the negative interactions among subgoals is to consider the interactions among the actions supporting such subgoals in our relaxed plans, and locate the possible reason for such interactions to penalize them. Interactions could arise because our relaxed plan computation

```
Procedure NEGFAM(G, Rp_G, g', a_g')
    cost_1 ← 0, cost_2 ← 0
    P_C ← 0, maxCost ← 0
    for (g_i ∈ G)
        a_i ← getSupportingAction(g_i, Rp_G)
                    cost_1                          ←
competingNeedsCost(g_i, a_i, g', a_g')
        cost_2 ← interferenceCost(g_i, a_i, g', a_g')
        maxCost ← max(cost_1, cost_2)
        if(maxCost > P_C)
            P_C ← maxCost
    end for
    return P_C;
End NEGFAM;
```

Figure 30. Interactions through actions

is greedy. It only considers the cheapest action[7] to support a given subgoal g', ignoring any negative interactions of the subgoal. Therefore, the intuition behind this idea is to adjust the residual cost returned by our relaxed plans, by assigning a penalty cost when interactions among their goal supporting actions are found, in order to get better estimates. We called this idea $NEGFAM$.[8]

$NEGFAM$ is also greedy because it only considers the actions directly supporting the subgoals in the relaxed plan, and it always keeps the interaction with maximum cost as its penalty cost. In case that there is no supporting action for a given subgoal g' (e.g. if $g' \in I$), the algorithm will take g' itself for comparison. $NEGFAM$ considers the following types of action interaction based on [Weld, 1999]:

[7] With respect to $C(a, l) + \mathcal{C}_a$

[8] Negative Factor By Actions: Max

1. Competing Needs: Two actions a and b have preconditions that are *statically* mutually exclusive, or at least one precondition of a is statically mutually exclusive with the subgoal g' given.

2. Interference: Two actions a and b, or one action a and a subgoal g' are interfering if the effect of a deletes b's preconditions, or a deletes g'.

Notice that we are only considering pairs of static mutex propositions when we do the action interaction analysis. The reason for this is that we just want to identify those preconditions that are critically responsible for the actions interactions, and give a penalty cost based on them. Once found a pair of static propositions, we have different ways of penalizing them. We show the description of the $NEGFAM$ technique on Figure 30. The procedure gets the current selected goals G, the relaxed plan Rp_G supporting them, and the subgoal g' being evaluated and action $a_{g'}$ supporting it. Then, it computes two different costs, one based on the competing needs of actions, and the second one based on their interference:

- For competing needs, we identify the proposition with maximum cost in the pair of static preconditions of the actions, and we set P_C to this cost. The idea is to identify what the minimum cost would be in order to support two competing preconditions. Given $p_1 \wedge p_2$, where $p_1 \in Prec(a_i)$ and $p_2 \in Prec(a_{g'})$, or $p_2 = g'$ when $\neg\exists_{a_{g'}}$, the cost is $cost_1 = max(C(p_1, levelo ff),C(p_2, levelo ff))$ if $lev(p_1 \wedge p_2) = \infty$. This penalty gets computed using the procedure $competingNeedsCost(g_i, a_i, g', a_{g'})$ in Figure 30.

- In case of interference, our penalty cost P_C is set to the cheapest alternate way (i.e. action) for supporting a proposition being deleted. The idea is to identify what the

additional cost would be in order to restore a critical precondition, which needs to be deleted to achieve another subgoal. Given $p_1 \in Prec(a_i)$, and $\neg p_1 \in Eff(a_{g'})$ or $g' = \neg p_1$, the cost is $cost_2 = min\{\mathcal{C}_x : \forall_x \; s.t \; p_1 \in Eff(x)\}$. This cost is computed using the procedure $InterferenceCost(g_i, a_i, g', a_{g'})$.

Our algorithm then selects the cost that maximizes our return value P_C given by the two techniques mentioned above. Our P_C is then added to the residual cost of subgoal g'.

Following our example 2 (Figure 28), we already mentioned that if we chose $g_3 = rock_2$ we would also select $g_1 = soil_1$ and $g_2 = rock_1$ in our original algorithm, which is not feasible. However, by taking into account the negative interactions among subgoals with $NEGFAM$ we would discard such unfeasible sets. For example, suppose that $G = \{g_3\}$ and $Rp_G = \{travel_{0,2}, \; sample_{rock_2,2}, \; comm_{rock_2,2,0}\}$, and the goal being considered for inclusion is $g' = g_1$ with residual cost 19, corresponding to its relaxed plan $Rp_{g'} = \{travel_{0,1}, \; sample_{soil_1,1}, \; comm_{rock_1,1,0}\}$. Notice that the supporting actions for g_3 and g_1 are $comm_{rock_2,2,0}$ and $comm_{soil_1,1,0}$ respectively. These actions have competing needs, one action requires the rover to be at $waypoint2$ while the other one assumes the rover is at $waypoint1$. The penalty cost P_C given by $NEGFAM$ for ignoring such interaction is 10, which is the maximum cost among the static mutex preconditions. Adding this value to our residual cost gives us a final cost of 29, which precludes the algorithm for considering g_1 (i.e. benefit = 20 - 29). Although $NEGFAM$ is also greedy since it may over increase the residual cost of a subgoal g', it improves over our original algorithm and $NEGFM$, returning better quality solutions for problems with complex interactions (as will be shown in our next section).

5.5. Empirical Evaluation

In the foregoing, We have described with illustrative examples, how complex interactions may affect the goal set selection process of $AltAlt^{ps}$. Our aim in this section is to show that planning graph reachability heuristics augmented with mutex analysis still provide efficient estimates for solving the PSP NET BENEFIT problem in the presence of complex goal interactions.

Since there are no benchmark PSP problems, we used existing STRIPS planning domains from the 2002 International Planning Competition [Long and Fox, 2003], and modified them to support explicit declaration of goal utilities and action costs. [9] In particular, our experiments include the domains of DriverLog and Rover. For the DriverLog domain, goal utilities ranged from 40 to 100, while the costs of the actions ranged from 3 to 70. Goal interactions were increased by considering bigger action execution costs, and modified paths in the network that the planning agent has to traverse. The idea was to place the most rewarding goals in the costlier paths of the network in order to increase the complexity of finding the most beneficial subset of goals. For the Rover domain, utilities ranged from 20 to 30, and action execution costs ranged from 4 to 45. In addition to the modifications introduced in the DriverLog domain to increase the level of interactions among goals, the Rover domain also allows for dead-ends, and loops in the network that the rover has to traverse. The idea was to present more options for the planning agent to fail. Consequently, it proved to be much more difficult to solve restricting even more the

[9] For a description of the domains and problems see Appendix A.

attainability of multiple goal sets. The design of this domain was inspired by the Rover domain presented by Smith(2004), without considering resources in our domain description.

We compared our new approach *AltWlt* to its predecessor (*AltAlt*^{ps}), and *Sapa*^{ps} [van den Briel *et al.*, 2004a]. Although *Sapa*^{ps} also uses planning graph heuristics to rank their goals, it does not provide mutex analysis and its search algorithm is different. We considered it pertinent to take into account both planners to see more clearly the impact of the techniques introduced by this dissertation. See the next section for further discussion on *Sapa*^{ps} [Do and Kambhampati, 2004].

We have also included in this section a run of *OptiPlan* [van den Briel *et al.*, 2004a] in the Rover domain, to demonstrate that our greedy approach is able to return high quality plans. *OptiPlan* is a planner that builds on the work of solving planning problems through IP [Vossen *et al.*, 1999], which generates plans that are optimal for a given plan length.[10] We did not compare to the approach presented by Smith(2004) because his approach was not yet available by the time of this writing.[11] All four planners were run on a P4 2.67Ghz CPU machine with 1.0GB RAM.

Figure 31(a) shows our results in the DriverLog and Rover domains. We see that *AltWlt* outperforms *AltAlt*^{ps} and *Sapa*^{ps} in most of the problems, returning higher quality solutions. In fact, it can be seen that *AltWlt* returns 13 times as much net benefit on average than *AltAlt*^{ps} in the DriverLog domain (i.e., a 1300% benefit increase). A similar scenario occurs with *Sapa*^{ps} , where *AltWlt* returns 1.42 times as much more benefit on average (a 42% benefit increase). A similar situation occurs with the Rover domain in Figure 31 (b),

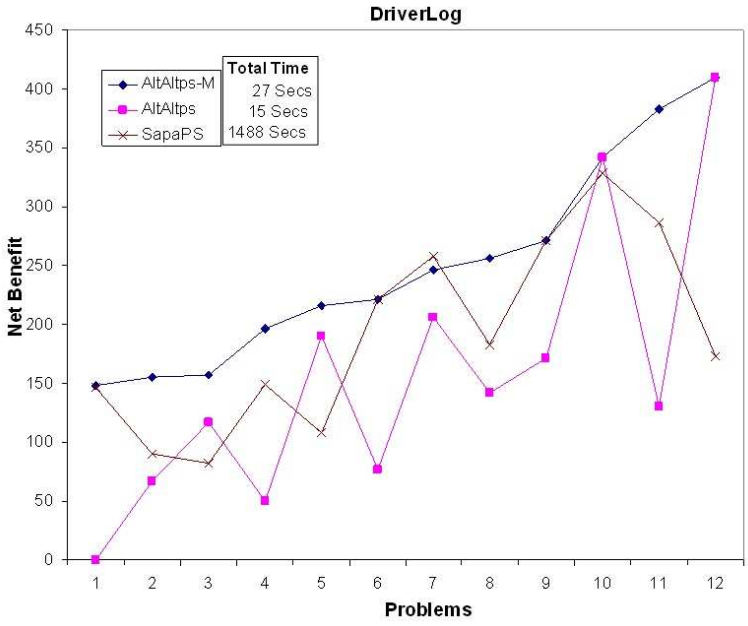
[10] For a more comprehensive description on *OptiPlan* see [van den Briel *et al.*, 2004a].

[11] Smith's approach takes as input a non-standard PDDL language, without the explicit representation of the operators descriptions.

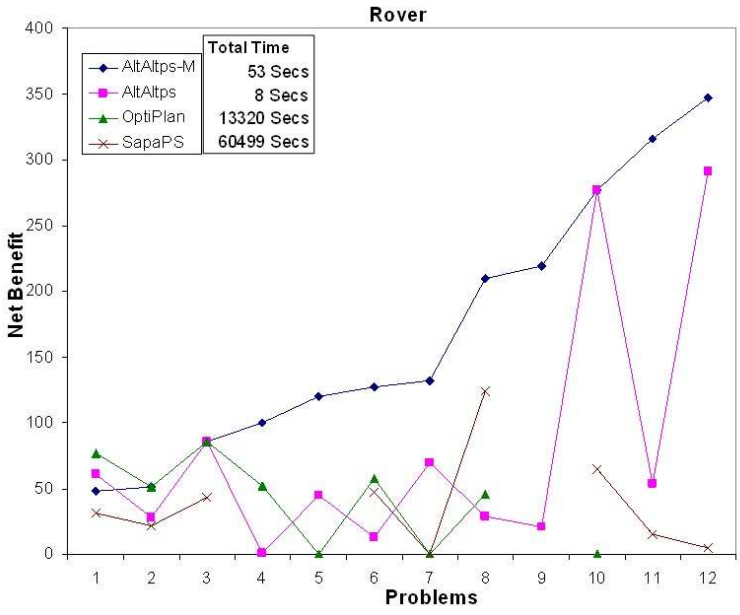
in which *AltWlt* returns 10 and 12 times as much more benefit on average than $AltAlt^{ps}$ and $Sapa^{ps}$ respectively. This corresponds to a 1000% and 1200% benefit increase over them. Although *OptiPlan* should in theory return optimal solutions for a given length, it is not able to scale up, reporting only upper bounds on most of its solutions. Furthermore, notice also in the plots that the total running time taken by *AltWlt* incurs a very little additional overhead over $AltAlt^{ps}$, and it is completely negligible in comparison to $Sapa^{ps}$ or *OptiPlan*.

Looking at the run times, it could appear at first glance that the set of problems are relatively easy to solve given the total accumulated time of $AltAlt^{ps}$. However, remember that for many classes of PSP problems, a trivially feasible, but decidedly non-optimal solution would be the "null" plan, and $AltAlt^{ps}$ is in fact returning faster but much lower quality solutions. We can see that the techniques introduced in *AltWlt* are helping the approach to select better goal sets by accounting for interactions. This is not happening in $AltAlt^{ps}$, where the goal sets returned are very small and easier to solve.

We also tested the performance of *AltWlt* in problems with less interactions. Specifically, we solved the suite of random problems from [van den Briel *et al.*, 2004a], including the ZenoTravel and Satellite planning domains [Long and Fox, 2003]. Although the gains there were less impressive, *AltWlt* was able to produce in general better quality solutions than the other approaches, returning bigger total net benefits.



(a) Cost-based Driverlog domain



(b) Cost-based Rover domain

Figure 31. Plots showing the total time and net benefit obtained by different PSP approaches

CHAPTER 6

Related Work

As discussed in previous chapters, the advancement in planning is mainly due to the extraction of heuristics from the problem representation. Heuristic state-space planning is one of the most successful plan synthesis approaches. In consequence, most effective and current planning systems are based on a combination of state-space search and planning graph related heuristics. One main difference of our approach with respect to anyone else is that our heuristic framework encodes positive as well as negative interactions among subgoals, improving the quality of our solutions. We will discuss in this Chapter the connections of our work to alternative approaches, and state-of-the-art plan synthesis algorithms.

In the first section, we will discuss and survey related state-space planning algorithms, emphasizing the main differences with respect to our approach, and possible venues for improving our work. Then, we will explore work on parallel planning, highlighting the limitations of competing approaches with respect to $AltAlt^p$. Finally, we will conclude this Chapter by providing a background on over-subscription planning, and alternative approaches to PSP Net Benefit.

6.1. Heuristic State-space Search and Disjunctive Planning

As we have already discussed in chapter 3, *AltAlt* has obvious rich connections to the existing work on *Graphplan* [Blum and Furst, 1997; Long and Fox, 1999] and regression state-space search planners [Bonet *et al.*, 1997; Bonet and Geffner, 1999; McDermott, 1999]. The closest planning algorithm to our approach is HSP-r [Bonet and Geffner, 1999]. As mentioned before, HSP-r is a regression planner that mostly uses the h_{add} heuristic to rank its search states. One of the main problems with such a heuristic is that it assumes goal independence, thrashing badly in problems with high number of interactions. Our approach however is able to extend on such ideas by considering positive as well as negative interactions using relaxed plans and penalty costs. Moreover, the idea of using the planning graph to select action instances and focus the regression search improves our approach even further over HSP-r. This idea is similar to the RIFO [Koehler, 1999] and *helpful actions* [Hoffmann and Nebel, 2001] techniques, that use relevance analysis to focus progression search.

Although *AltAlt* has proven to be efficient and effective in a wide spectrum of planning problems, it lacks behind current state-of-the-art planning systems. Most successful planning systems are currently based on some sort of forward state-space algorithm enhanced with different planning graph heuristic relaxations, as corroborated by the most recent international planning competitions [Long and Fox, 2003; Hoffmann and Edelkamp, 2005]. Although progression search tends to have a bigger branching factor than regression search, the states produced by progression search are complete and consistent, which in turn results in more accurate heuristic estimates. Moreover, the relevance analysis used by

progression search algorithms helps them to reduce their branching factor improving their overall efficiency.

One of such planning systems is FF [Hoffmann and Nebel, 2001]. FF's base heuristic can be seen as a graphplan relaxation. During its forward search, it computes from scratch a relaxed plan for each state visited. Its relaxed plan heuristic takes into account positive interactions among subgoals, and in that sense is similar to the $h_{AdjSum2M}$ heuristic of *AltAlt*. However, negative interactions are still ignored. FF search strategy is a type of enforced hill-climbing, combining local search and systematic search. The planner uses the *helpful actions* technique from its relaxed plans to cut out search branches. In that sense, it is similar to *AltAlt* using the *PACTION* procedure directly from the planning graph.

LPG [Gerevini and Serina, 2002] has been another very successful planning system that also uses subgraphs from the planning graph data structure to guide its search. The operators for moving from one search state into another are based on modifications to the subgraphs. In that sense, its heuristics differ from the distance-based estimations discussed in this Dissertation given that they weight how costly is to repair the inconsistencies found in a particular subgraph.

As discussed in [Nguyen and Kambhampati, 2000; Nguyen *et al.*, 2002], there are also several rich connections between our strategies for deriving the heuristics from the planning graphs, and recent advances in heuristic search, such as pattern databases [Culberson and Schaeffer, 1998], capturing subproblem interactions [Korf and Taylor, 1996; Korf, 1997], and greedy regression tables [Refanidis and Vlahavas, 2001] by the GRT planning system. It is worth to mention that the GRT planning system implements a progression search algorithm, but it only computes its heuristic once backwards from the top level goals as a preprocessing

step. In order to construct the heuristic backwards, the actions of the problem have to be inverted, enhancing the top level goal set with the missing propositions to obtain better estimates for its tables, and discarding objects that are not relevant. Notice that in our case a similar idea could be tried during the regression search of *AltAlt*. Remember that the states resulting from regression search constitute partial assignments (i.e, they represent a set of states due to the missing propositions). In consequence, we could try to enhance (or complete) our regressed states to also improve our heuristics during search.

Finally, one of the most recent and successful planners is Fast Downward [Helmert and Richter, 2004; Helmert, 2004]. Motivated by the fact that progression planners tend to suffer in domains with deadends or frequent plateau regions (since most of them ignore negative interactions among subgoals), the Fast Downward algorithm introduced multi-value state variables as a natural way of computing and compactly representing subgoals and their interactions. Fast Downward uses hierarchical decompositions of planning tasks based on causal graphs for computing its heuristic function. The main idea of the heuristic is to split the planning task, and combine the lengths of the solutions of the parts to form a distance heuristic for the overall task. The heuristic estimates the cost of changing the values of the state variables in the subtasks. In that sense, it is similar to LPG, which tries to repair locally the inconsistencies of its subgraphs.

Although *AltAlt* lacks behind the current state-of-the-art planning systems, it still remains as one of the best regression state-space search planners up to date. Moreover, some of the new techniques employed by progression planners could be extended to improve the two phases of our approach, the heuristics and the search process. For example, the multi-value state variable representation of Fast Downward could be adapted to improve

the planning graph expansion phase of *AltAlt*. Moreover, the use of *helpful actions* and consistency enforcement techniques, such as those described in [Hoffmann and Nebel, 2001; Gerevini and Schubert, 1998; Fox and Long, 1998; Rintanen, 2000], could be tried to further improve the backward search algorithm of *AltAlt*.

6.2. State-space Parallel Planning and Heuristics

The idea of partial exploration of parallelizable sets of actions is not new [Kabanza, 1997; Godefroid and Kabanza, 1991; Do and Kambhampati, 2001]. It has been studied in the area of concurrent and reactive planning, where one of the main goals is to approximate optimal parallelism. However, most of the research there has been focused on forward chaining planners [Kabanza, 1997], where the state of the world is completely known. It has been implied that backward-search methods are not suitable for this kind of analysis [Godefroid and Kabanza, 1991] because the search nodes correspond to partial states. We have shown in Chapter 4 that backward-search methods can also be used to approximate parallel plans in the context of classical planning.

Optimization of plans according to different criteria (e.g. execution time, quality, etc) has also been done as a post-processing step. The post-processing computation of a given plan to maximize its parallelism has been discussed by Backstrom(1998). Reordering and de-ordering techniques are used to maximize the parallelism of the plan. In de-ordering techniques ordering relations can only be removed, not added. In reordering, arbitrary modifications to the plan are allowed. In the general case this problem is NP-Hard and it is difficult to approximate [Backstrom, 1998]. Furthermore, as discussed in Section 4.1,

post-processing techniques are just concerned with modifying the order between the existing actions of a given plan. Our approach not only considers modifying such orderings but also inserting new actions online which can minimize the possible number of parallel steps of the problem.

We have already discussed *Graphplan* based planners [Long and Fox, 1999; Kautz and Selman, 1999], which return optimal plans based on the number of time steps. As mentioned in Chapter 4, *Graphplan* uses IDA* to include the greatest number of parallel actions at each time step of the search. However, this iterative procedure is very time consuming and it does not provide any guarantee on the number of actions in its final plans. There have been a few attempts to minimize the number of actions in these planners [Huang *et al.*, 1999] by using some domain control knowledge based on the generation of rules for each specific planning domain. The *Graphplan* algorithm tries to maximize its parallelism by satisfying most of the subgoals at each time step, if the search fails then it backtracks and reduces the set of parallel actions being considered one level before. $AltAlt^p$ does the opposite, it tries to guess initial parallel nodes given the heuristics, and iteratively adds more actions to these nodes as possible with the *PushUp* procedure later during search.

More recently, there has been some work on generalizing forward state-space search to handle action concurrency in metric temporal domains. Of particular relevance to this work are the temporal TLPlan [Bacchus and Ady, 2001] and SAPA [Do and Kambhampati, 2001] planners. Both of these planners are designed specifically for handling metric temporal domains, and use similar search strategies. The main difference between them being that Temporal TLPlan depends on hand-coded search control knowledge to guide its search, while SAPA (like $AltAlt^p$) uses heuristics derived from (temporal) planning graphs. As

such, both these planners can be co-opted to produce parallel plans in classical domains. Both these planners do a forward chaining search, and like *AltAlt*^{p}, both of them achieve concurrency incrementally, without projecting sets of actions, in the following way. Normal forward search planners start with the initial state S_0, corresponding to time t_0, consider all actions that apply to S_0, and choose one, say a_1 apply it to S_0, getting S_1. They simultaneously progress the "system clock" from t_0 to t_1. In order to allow for concurrency, the planners by Bacchus and Ady, and Do and Kambhampati essentially decouple the "action application" and "clock progression." At every point in the search, there is a non-deterministic choice - between progressing the clock, or applying (additional) actions at the current time point. From the point of view of these planners, *AltAlt*^{p} can be seen as providing heuristic guidance for this non-deterministic choice (modulo the difference that *AltAlt*^{p} does regression search). The results of empirical comparisons between *AltAlt*^{p} and SAPA, which show that *AltAlt*^{p} outperforms SAPA, suggest that the heuristic strategies employed in *AltAlt*^{p} including the incremental fattening, and the pushup procedure, can be gainfully adapted to these planners to increase the concurrency in the solution plans. Finally, HSP*, and TP4, its extension to temporal domains, are both heuristic state-space search planners using regression that are capable of producing parallel plans [Haslum and Geffner, 2000]. TP4 can be seen as the regression version of the approach used in SAPA and temporal TLPlan. Our experiments however demonstrate that neither of these planners scales well in comparison to *AltAlt*^{p}.

The *PushUp* procedure can be seen as a plan compression procedure. As such, it is similar to other plan compression procedures such as "double-back optimization" [Crawford, 1996]. One difference is that double-back is used in the context of a local search, while

PushUp is used in the context of a systematic search. Double-back could be also applied to any finished plan or schedule, but as any other post-processing approach its outcome would depend highly on the plan given as input. We can conclude that our approach $AltAlt^p$ remains as the state-of-the-art in the generation of parallel plans through state-space search.

6.3. Heuristic Approaches to Over-subscription Planning

As mentioned in Chapter 5, there has been very little work on PSP in planning. One possible exception is the PYRRHUS planning system [Williamson and Hanks, 1994] which considers an interesting variant of the partial satisfaction planning problem. In PYRRHUS, the quality of the plans is measured by the utilities of the goals and the amount of resource consumed. Utilities of goals decrease if they are achieved later than the goals' deadlines. Unlike the PSP problem discussed in this Thesis, all the logical goals still need to be achieved by PYRRHUS for the plan to be valid. It would be interesting to extend the PSP model to consider degree of satisfaction of individual goals.

More recently, Smith (2003) motivated oversubscription problems in terms of their applicability to the NASA planning problems. Smith (2004) also proposed a planner for oversubscription in which the solution of the abstracted planning problem is used to select the subset of goals and the orders to achieve them. The abstract planning problem is built by propagating the cost on the planning graph and constructing the *orienteering* problem. The goals and their orderings are then used to guide a POCL planner. In this sense, this approach is similar to $AltAlt^{ps}$; however, the orienteering problem needs to be constructed using domain-knowledge for different planning domains. Smith (2004) also speculated that

planning-graph based heuristics are not particularly suitable for PSP problems where goals are highly interacting. His main argument is that heuristic estimates derived from planning graphs implicitly make the assumption that goals are independent. However, as shown in this Dissertation, reachability estimates can be improved using the mutex information also contained in planning graphs, allowing us to solve problems with complex goal interactions.

Probably the most obvious way to optimally solve the PSP NET BENEFIT problem is by modeling it as a fully-observable Markov Decision Process (MDP) [Hoey *et al.*, 1999] with a finite set of states. MDPs naturally permit action cost and goal utilities, but we found in our studies that an MDP based approach for the PSP NET BENEFIT problem appears to be impractical, even the very small problems generate too many states. To prove our assumption, we modeled a set of test problems as MDPs and solved them using SPUDD [Hoey *et al.*, 1999].[1] SPUDD is an MDP solver that uses value iteration on algebraic decision diagrams, which provides an efficient representation of the planning problem. Unfortunately, SPUDD was not able to scale up, solving only the smallest problems.[2]

Remember also that PSP problems could be solved optimally using Integer Programming techniques [Haddawy and Hanks, 1993]. An approach included in our evaluation section is *OptiPlan*. *OptiPlan* is a planning system that provides an extension to the state change programming (IP) model by Vossen *et al.*(1999). The main extension in *OptiPlan* is the use of Graphplan [Blum and Furst, 1997] to eliminate many unnecessary variables. Unfortunately, the current version of *OptiPlan* does not scale up well to bigger sized problems.

[1] We thank Will Cushing and Menkes van den Briel who first suggested the MDP modeling idea.
[2] Details on the MDP model and results can be found in [van den Briel *et al.*, 2005].

The only other heuristic state-space based approach to PSP is $Sapa^{ps}$ [Do and Kambhampati, 2004; van den Briel *et al.*, 2004a]. Unlike to *AltWlt*, $Sapa^{ps}$ does not select a subset of the goals up front, but uses an anytime A* heuristic search framework in which goals are treated as "soft constraints" to select them during planning. Any executable plan is considered a potential solution, with the quality of the plan measured in terms of its net benefit. $Sapa^{ps}$ is a forward state space planner, which does not include mutex analysis in its search framework.

Over-subscription issues have received relatively more attention in the scheduling community. Earlier work in over-subscription scheduling used greedy approaches, in which tasks of higher priorities are scheduled first [Kramer and Giuliano, 1997; Potter and Gasch, 1998]. The approach used by *AltWlt* is more sophisticated in that it considers the residual cost of a subgoal in the context of an existing partial plan for achieving other selected goals, taking into account complex interactions among the goals. More recent efforts have used stochastic greedy search algorithms on constraint-based intervals [Frank *et al.*, 2001], genetic algorithms [Globus *et al.*, 2003], and iterative repairing technique [Kramer and Smith, 2003] to solve this problem more effectively.

CHAPTER 7

Concluding Remarks

The current work is motivated by the need of domain independent heuristic estimators by plan synthesis algorithms. Heuristic functions that can be extracted automatically from the problem representation, and that can be used across different planning problems and domains. Particularly, we have shown in this Dissertation, that state-space planning can be more efficient and effective in solving more complex problems when enhanced with heuristic functions. A novel heuristic framework was developed based on the reachability information encoded in planning graphs. It was shown that distance-based heuristics extracted from planning graphs are powerful approximations for a variety of planning problems.

More specifically, techniques for controlling the cost of computing the heuristics and reducing the branching factor of the search were introduced. Planning graph heuristics were also developed for parallel state-space planning, where an online plan compression algorithm was designed to improve even further the quality of the solutions returned by the system. The empirical evaluation showed that the greedy approach was an attractive tradeoff between quality and efficiency in the generation of parallel plans.

The portability of the heuristic estimates was again demonstrated when we adjusted state-space planning to account for over-subscription planning problems. A greedy algorithm was developed that is able to select a subset of the top-level goals upfront using planning graph estimates. It was also shown, that the mutex analysis from the planning graph is a rich source of information for improving even further the heuristics, addressing more complex planning problems.

Finally, extensive empirical analysis on each different application of state-space planning was presented at each chapter, highlighting the heuristics used by the algorithms. Related work on heuristic planning was also discussed at the end of the dissertation.

7.1. Future Work

The solutions presented by this work are based on state-space planning, more specifically regression planning. Although, regression state-space planning considers relevance of actions to reduce the branching factor of the search, this one can still be quite large, producing many inconsistent states. A better alternative would be to consider *lifted* planning representations, in which states are partially instantiated. One of the main advantages of lifting is the reduction of the planner's search space by delaying the introduction of ground atoms into the plan, thus avoiding the large branching factor of planning problems [Yang and Chan, 1994; Younes and Simmons, 2002]. In consequence, lifting avoids the problem of excessive memory usage [Ginsberg and Parkes, 2000; Parkes, 1999] by pruning the search space. However, there does not yet exist a convincing approach for planning with lifted actions. This is mainly due to the lack of heuristics for

partially instantiated states, and the overhead for maintaining consistency on the bindings of the lifted operators. Furthermore, the indexing and caching schemas that are essential to the speed of ground actions [Parkes, 1999] may not be directly applicable to lifted representations.

A lifted state would have some variables and certain domains of values over them. In fact, a partially instantiated state would represent a set of states, each one would correspond to an assignment of the values to the free variables in the state. The challenge would be to adjust the reachability information provided by planning graphs to cover most of the variables in the state, without enumerating all different instantiations. One simple, but naive idea would be to consider the minimum cost (e.g., level information) in the planning graph among all different instantiations of a particular variable. However, as mentioned before, this heuristic would be too costly to compute since we would need to enumerate all possible combinations of values for each variable. A better tradeoff would be to consider distance as well as cover information for each particular variable. In other words, issues like: variables appearing more in a state, cardinality of the variables (i.e., size of their domains), and reachability information may be critical to compute good heuristic estimates. Future work will try to shed light on these issues, understanding that lifting could be of great help for those particular planning problems (e.g., probabilistic planning, temporal planning, planning under uncertainty, etc) where the branching factor of the search is a real and complicated problem.

As mentioned in Chapter 1, taking into account complex goal interactions to compute admissible heuristics remains a challenging problem. Another future direction would be to consider positive as well as negative goal interactions in the application of distance-

based heuristics in other critical planning frameworks, including among these temporal planning and least commitment planning where negative interactions are mostly ignored. The effectiveness of considering positive interactions by the relax plan heuristic has been widely shown [Hoffmann and Nebel, 2001; Nguyen *et al.*, 2002; Gerevini and Serina, 2002], and we believe that there are problems in which negative interactions play a crucial role, but, unfortunately, no planning system has addressed such issue at all. Planning graphs provide a natural way of considering strong interactions, and we plan to further exploit them, in other more complex planning frameworks.

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APPENDIX A

PSP DOMAIN DESCRIPTIONS AND COST-UTILITY PROBLEMS

A.1. Rover Domain

```
(define (domain rover) (:requirements :strips) (:predicates
    (at ?x ?y)
    (atlander ?x ?y)
    (cantraverse ?r ?x ?y)
    (equippedforsoilanalysis ?r)
    (equippedforrockanalysis ?r)
    (equippedforimaging ?r) (empty ?s)
    (haverockanalysis ?r ?w)
    (havesoilanalysis ?r ?w)
    (full ?s)
    (calibrated ?c ?r ?o ?p)
    (supports ?c ?m)
    (available ?r)
    (visible ?w ?p)
    (haveimage ?r ?o ?m)
    (communicatedsoildata ?w)
    (communicatedrockdata ?w)
    (communicatedimagedata ?o ?m)
    (atsoilsample ?w)
    (atrocksample ?w)
    (visiblefrom ?o ?w)
    (storeof ?s ?r)
    (calibrationtarget ?i ?o)
    (onboard ?i ?r)
    (channelfree ?l)
    (rover ?x)
    (waypoint ?x)
    (store ?x)
    (camera ?x)
    (mode ?x)
    (lander ?x)
```

```
    (objective ?x))

(:action navigate
 :parameters ( ?x ?y ?z)
 :precondition
    (and (rover ?x) (waypoint ?y) (waypoint ?z)
    (cantraverse ?x ?y ?z) (available ?x)
    (at ?x ?y) (visible ?y ?z))
 :effect
    (and (not (at ?x ?y)) (at ?x ?z)))

(:action samplesoil
 :parameters ( ?x ?s ?p)
 :precondition
    (and (rover ?x) (store ?s) (waypoint ?p)
    (at ?x ?p) (atsoilsample ?p)
    (equippedforsoilanalysis ?x) (storeof ?s ?x) (empty ?s))
 :effect
    (and (not (empty ?s)) (not (atsoilsample ?p)) (full ?s) (havesoilanalysis ?x ?p)))

(:action samplerock
 :parameters ( ?x ?s ?p)
 :precondition
    (and (rover ?x) (store ?s) (waypoint ?p)
    (at ?x ?p) (atrocksample ?p)
    (equippedforrockanalysis ?x) (storeof ?s ?x) (empty ?s))
 :effect
    (and (full ?s) (haverockanalysis ?x ?p) (not (empty ?s)) (not (atrocksample ?p))))

(:action drop
 :parameters ( ?x ?y)
 :precondition
```

```
    (and (rover ?x) (store ?y)
    (storeof ?y ?x) (full ?y))
 :effect
    (and (empty ?y) (not (full ?y))))

(:action calibrate
 :parameters ( ?r ?i ?t ?w)
 :precondition
    (and (rover ?r) (camera ?i) (objective ?t) (waypoint ?w)
    (equippedforimaging ?r)
    (calibrationtarget ?i ?t)
    (at ?r ?w) (visiblefrom ?t ?w) (onboard ?i ?r))
 :effect
     (calibrated ?i ?r ?t ?w))

(:action takeimage
 :parameters ( ?r ?p ?o ?i ?m)
 :precondition
    (and (rover ?r) (waypoint ?p) (objective ?o) (camera ?i) (mode ?m)
    (calibrated ?i ?r ?o ?p) (onboard ?i ?r) (equippedforimaging ?r) (supports ?i ?m)
    (visiblefrom ?o ?p) (at ?r ?p))
 :effect
    (and (haveimage ?r ?o ?m) (not (calibrated ?i ?r ?o ?p))))

(:action communicatesoildata
 :parameters ( ?r ?l ?p ?x ?y)
 :precondition
    (and (rover ?r) (lander ?l) (waypoint ?p) (waypoint ?x) (waypoint ?y)
    (at ?r ?x) (atlander ?l ?y)
    (havesoilanalysis ?r ?p)
    (visible ?x ?y)
    (available ?r) (channelfree ?l))
```

```
 :effect
      (communicatedsoildata ?p))

(:action communicaterockdata
 :parameters ( ?r ?l ?p ?x ?y)
 :precondition
    (and (rover ?r) (lander ?l) (waypoint ?p) (waypoint ?x) (waypoint ?y)  (at ?r ?x)
    (atlander ?l ?y) (haverockanalysis ?r ?p) (visible ?x ?y) (available ?r) (channelfree ?l))
 :effect
    (communicatedrockdata ?p))

(:action communicateimagedata
 :parameters ( ?r ?l ?o ?m ?x ?y)
 :precondition
    (and (rover ?r) (lander ?l) (objective ?o) (mode ?m) (waypoint ?x) (waypoint ?y)
    (at ?r ?x) (atlander ?l ?y) (haveimage ?r ?o ?m) (visible ?x ?y) (available ?r) (channelfree ?l))
 :effect
    (communicatedimagedata ?o ?m))

)
```

A.2. Rover Problem

A.2.1. Problem 11.

```
(define (problem roverprob11) (:domain Rover) (:objects
    general colour highres lowres rover0 rover0store
    waypoint0 waypoint1 waypoint2 waypoint3 waypoint4
    waypoint5 waypoint6 waypoint7 waypoint8 waypoint9
    waypoint10 waypoint11 waypoint12 waypoint13 waypoint14
    waypoint15 waypoint16 waypoint17 waypoint18 waypoint19
    waypoint20  waypoint21
    camera0 camera1 camera2 camera3 camera4 camera5 camera6
```

```
    camera7 camera8
    objective0 objective1 objective2 objective3 objective4
    objective5 objective6 objective7 objective8)
(:init
    (visible waypoint0 waypoint5)
    (visible waypoint5 waypoint0)
    (visible waypoint0 waypoint8)
    (visible waypoint8 waypoint0)
    (visible waypoint0 waypoint12)
    (visible waypoint12 waypoint0)
    (visible waypoint0 waypoint13)
    (visible waypoint13 waypoint0)
    (visible waypoint0 waypoint18)
    (visible waypoint18 waypoint0)
    (visible waypoint0 waypoint21)
    (visible waypoint21 waypoint0)
    (visible waypoint1 waypoint2)
    (visible waypoint2 waypoint1)
    (visible waypoint1 waypoint6)
    (visible waypoint6 waypoint1)
    (visible waypoint2 waypoint1)
    (visible waypoint1 waypoint2)
    (visible waypoint2 waypoint6)
    (visible waypoint6 waypoint2)
    (visible waypoint3 waypoint2)
    (visible waypoint2 waypoint3)
    (visible waypoint4 waypoint3)
    (visible waypoint3 waypoint4)
    (visible waypoint4 waypoint21)
    (visible waypoint21 waypoint4)
    (visible waypoint5 waypoint4)
    (visible waypoint4 waypoint5)
```

```
(visible waypoint5 waypoint6)
(visible waypoint6 waypoint5)
(visible waypoint6 waypoint12)
(visible waypoint12 waypoint6)
(visible waypoint7 waypoint8)
(visible waypoint8 waypoint7)
(visible waypoint7 waypoint11)
(visible waypoint11 waypoint7)
(visible waypoint8 waypoint12)
(visible waypoint12 waypoint8)
(visible waypoint9 waypoint8)
(visible waypoint8 waypoint9)
(visible waypoint9 waypoint13)
(visible waypoint13 waypoint9)
(visible waypoint10 waypoint9)
(visible waypoint9 waypoint10)
(visible waypoint10 waypoint11)
(visible waypoint11 waypoint10)
(visible waypoint13 waypoint12)
(visible waypoint12 waypoint13)
(visible waypoint13 waypoint14)
(visible waypoint14 waypoint13)
(visible waypoint13 waypoint17)
(visible waypoint17 waypoint13)
(visible waypoint14 waypoint15)
(visible waypoint15 waypoint14)
(visible waypoint15 waypoint16)
(visible waypoint16 waypoint15)
(visible waypoint15 waypoint19)
(visible waypoint19 waypoint15)
(visible waypoint16 waypoint17)
(visible waypoint17 waypoint16)
```

```
(visible waypoint17 waypoint18)
(visible waypoint18 waypoint17)
(visible waypoint18 waypoint19)
(visible waypoint19 waypoint18)
(visible waypoint18 waypoint21)
(visible waypoint21 waypoint18)
(visible waypoint19 waypoint20)
(visible waypoint20 waypoint19)
(visible waypoint19 waypoint21)
(visible waypoint21 waypoint19)
(visible waypoint20 waypoint21)
(visible waypoint21 waypoint20)
(lander general)
(mode colour)
(mode highres)
(mode lowres)
(waypoint waypoint0)
(waypoint waypoint1)
(atsoilsample waypoint1)
(waypoint waypoint2)
(atsoilsample waypoint2)
(atrocksample waypoint2)
(waypoint waypoint3)
(atrocksample waypoint3)
(waypoint waypoint4)
(atsoilsample waypoint4)
(atrocksample waypoint4)
(waypoint waypoint5)
(atrocksample waypoint5)
(atsoilsample waypoint5)
(waypoint waypoint6)
(atrocksample waypoint6)
```

```
(atsoilsample waypoint6)
(waypoint waypoint7)
(atsoilsample waypoint7)
(waypoint waypoint8)
(atsoilsample waypoint8)
(atrocksample waypoint8)
(waypoint waypoint9)
(atrocksample waypoint9)
(waypoint waypoint10)
(atsoilsample waypoint10)
(atrocksample waypoint10)
(waypoint waypoint11)
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(atsoilsample waypoint11)
(waypoint waypoint12)
(atrocksample waypoint12)
(atsoilsample waypoint12)
(waypoint waypoint13)
(atsoilsample waypoint13)
(atrocksample waypoint13)
(waypoint waypoint14)
(atsoilsample waypoint14)
(atrocksample waypoint14)
(waypoint waypoint15)
(atrocksample waypoint15)
(atsoilsample waypoint15)
(waypoint waypoint16)
(atsoilsample waypoint16)
(waypoint waypoint17)
(atrocksample waypoint17)
(atsoilsample waypoint17)
(waypoint waypoint18)
```

```
(atrocksample waypoint18)
(atsoilsample waypoint18)
(waypoint waypoint19)
(atsoilsample waypoint19)
(atrocksample waypoint19)
(waypoint waypoint20)
(atrocksample waypoint20)
(waypoint waypoint21)
(atsoilsample waypoint21)
(atrocksample waypoint21)
(atlander general waypoint0)
(channelfree general)
(rover rover0)
(store rover0store)
(at rover0 waypoint0)
(available rover0)
(storeof rover0store rover0)
(empty rover0store)
(equippedforsoilanalysis rover0)
(equippedforrockanalysis rover0)
(equippedforimaging rover0)
(cantraverse rover0 waypoint0 waypoint5)
(cantraverse rover0 waypoint0 waypoint12)
(cantraverse rover0 waypoint0 waypoint8)
(cantraverse rover0 waypoint0 waypoint13)
(cantraverse rover0 waypoint0 waypoint18)
(cantraverse rover0 waypoint0 waypoint21)
(cantraverse rover0 waypoint1 waypoint2)
(cantraverse rover0 waypoint1 waypoint6)
(cantraverse rover0 waypoint2 waypoint1)
(cantraverse rover0 waypoint2 waypoint3)
(cantraverse rover0 waypoint2 waypoint6)
```

```
(cantraverse rover0 waypoint3 waypoint2)
(cantraverse rover0 waypoint4 waypoint3)
(cantraverse rover0 waypoint4 waypoint21)
(cantraverse rover0 waypoint5 waypoint4)
(cantraverse rover0 waypoint5 waypoint6)
(cantraverse rover0 waypoint6 waypoint1)
(cantraverse rover0 waypoint6 waypoint2)
(cantraverse rover0 waypoint6 waypoint5)
(cantraverse rover0 waypoint6 waypoint12)
(cantraverse rover0 waypoint7 waypoint8)
(cantraverse rover0 waypoint7 waypoint11)
(cantraverse rover0 waypoint8 waypoint12)
(cantraverse rover0 waypoint8 waypoint7)
(cantraverse rover0 waypoint9 waypoint8)
(cantraverse rover0 waypoint9 waypoint13)
(cantraverse rover0 waypoint10 waypoint9)
(cantraverse rover0 waypoint10 waypoint11)
(cantraverse rover0 waypoint10 waypoint14)
(cantraverse rover0 waypoint11 waypoint7)
(cantraverse rover0 waypoint11 waypoint10)
(cantraverse rover0 waypoint13 waypoint12)
(cantraverse rover0 waypoint13 waypoint14)
(cantraverse rover0 waypoint13 waypoint17)
(cantraverse rover0 waypoint14 waypoint15)
(cantraverse rover0 waypoint15 waypoint14)
(cantraverse rover0 waypoint15 waypoint16)
(cantraverse rover0 waypoint15 waypoint19)
(cantraverse rover0 waypoint16 waypoint17)
(cantraverse rover0 waypoint16 waypoint15)
(cantraverse rover0 waypoint17 waypoint13)
(cantraverse rover0 waypoint17 waypoint18)
(cantraverse rover0 waypoint18 waypoint19)
```

```
(cantraverse rover0 waypoint18 waypoint21)
(cantraverse rover0 waypoint19 waypoint18)
(cantraverse rover0 waypoint19 waypoint20)
(cantraverse rover0 waypoint19 waypoint21)
(cantraverse rover0 waypoint20 waypoint21)
(camera camera0)
(onboard camera0 rover0)
(calibrationtarget camera0 objective0)
(supports camera0 colour)
(supports camera0 highres)
(supports camera0 lowres)
(objective objective0)
(visiblefrom objective0 waypoint19)
(camera camera1)
(onboard camera1 rover0)
(calibrationtarget camera1 objective1)
(supports camera1 colour)
(supports camera1 highres)
(supports camera1 lowres)
(objective objective1)
(visiblefrom objective1 waypoint21)
(camera camera2)
(onboard camera2 rover0)
(calibrationtarget camera2 objective2)
(supports camera2 colour)
(supports camera2 highres)
(supports camera2 lowres)
(objective objective2)
(visiblefrom objective2 waypoint4)
(visiblefrom objective2 waypoint5)
(visiblefrom objective2 waypoint3)
(camera camera3)
```

```
(onboard camera3 rover0)
(calibrationtarget camera3 objective3)
(supports camera3 colour)
(supports camera3 highres)
(supports camera3 lowres)
(objective objective3)
(visiblefrom objective3 waypoint5)
(visiblefrom objective3 waypoint6)
(visiblefrom objective3 waypoint4)
(visiblefrom objective3 waypoint0)
(camera camera4)
(onboard camera4 rover0)
(calibrationtarget camera4 objective4)
(supports camera4 colour)
(supports camera4 highres)
(supports camera4 lowres)
(objective objective4)
(visiblefrom objective4 waypoint12)
(camera camera5)
(onboard camera5 rover0)
(calibrationtarget camera5 objective5)
(supports camera5 colour)
(supports camera5 highres)
(supports camera5 lowres)
(objective objective5)
(visiblefrom objective5 waypoint8)
(visiblefrom objective5 waypoint9)
(camera camera6)
(onboard camera6 rover0)
(calibrationtarget camera6 objective6)
(supports camera6 colour)
(supports camera6 highres)
```

```
    (supports camera6 lowres)
    (objective objective6)
    (visiblefrom objective6 waypoint11)
    (visiblefrom objective6 waypoint7)
    (visiblefrom objective6 waypoint10)
    (camera camera7)
    (onboard camera7 rover0)
    (calibrationtarget camera7 objective7)
    (supports camera7 colour)
    (supports camera7 highres)
    (supports camera7 lowres)
    (objective objective7)
    (visiblefrom objective7 waypoint14)
    (visiblefrom objective7 waypoint15)
    (camera camera8)
    (onboard camera8 rover0)
    (calibrationtarget camera8 objective8)
    (supports camera8 colour)
    (supports camera8 highres)
    (supports camera8 lowres)
    (objective objective8)
    (visiblefrom objective8 waypoint17)
    (visiblefrom objective8 waypoint13)
) (:goal (and (communicatedsoildata waypoint1) (communicatedsoildata
waypoint2) (communicatedsoildata waypoint4) (communicatedsoildata
waypoint5) (communicatedsoildata waypoint6) (communicatedsoildata
waypoint7) (communicatedsoildata waypoint8) (communicatedsoildata
waypoint10) (communicatedsoildata waypoint11) (communicatedsoildata
waypoint12) (communicatedsoildata waypoint13) (communicatedsoildata
waypoint14) (communicatedsoildata waypoint15) (communicatedsoildata
waypoint16) (communicatedsoildata waypoint17) (communicatedsoildata
waypoint18) (communicatedsoildata waypoint19) (communicatedsoildata
```

```
waypoint21) (communicatedrockdata waypoint2) (communicatedrockdata
waypoint3) (communicatedrockdata waypoint4) (communicatedrockdata
waypoint5) (communicatedrockdata waypoint6) (communicatedrockdata
waypoint8) (communicatedrockdata waypoint9) (communicatedrockdata
waypoint10) (communicatedrockdata waypoint11) (communicatedrockdata
waypoint12) (communicatedrockdata waypoint13) (communicatedrockdata
waypoint14) (communicatedrockdata waypoint15) (communicatedrockdata
waypoint17) (communicatedrockdata waypoint18) (communicatedrockdata
waypoint19) (communicatedrockdata waypoint20) (communicatedrockdata
waypoint21) (communicatedimagedata objective0 highres)
(communicatedimagedata objective1 lowres) (communicatedimagedata
objective1 highres) (communicatedimagedata objective1 colour)
(communicatedimagedata objective2 highres) (communicatedimagedata
objective2 colour) (communicatedimagedata objective2 lowres)
(communicatedimagedata objective3 highres) (communicatedimagedata
objective4 lowres) (communicatedimagedata objective4 colour)
(communicatedimagedata objective4 highres) (communicatedimagedata
objective5 lowres) (communicatedimagedata objective5 colour)
(communicatedimagedata objective6 lowres) (communicatedimagedata
objective7 lowres) (communicatedimagedata objective7 colour)
(communicatedimagedata objective7 highres) (communicatedimagedata
objective8 lowres) ) ) )
```

A.2.2. Problem 11 Cost File and Graphical Representation.

```
374 navigate(rover0,waypoint0,waypoint5) 11
navigate(rover0,waypoint0,waypoint8) 29
navigate(rover0,waypoint0,waypoint12) 15
navigate(rover0,waypoint0,waypoint13) 27
navigate(rover0,waypoint0,waypoint18) 28
navigate(rover0,waypoint0,waypoint21) 21
calibrate(rover0,camera3,objective3,waypoint0) 1
```

navigate(rover0,waypoint5,waypoint4) 9

navigate(rover0,waypoint5,waypoint6) 9

navigate(rover0,waypoint8,waypoint7) 11

navigate(rover0,waypoint8,waypoint12) 24

navigate(rover0,waypoint13,waypoint12) 34

navigate(rover0,waypoint13,waypoint14) 19

navigate(rover0,waypoint13,waypoint17) 8

navigate(rover0,waypoint18,waypoint19) 10

navigate(rover0,waypoint18,waypoint21) 8

samplesoil(rover0,rover0store,waypoint5) 5

samplesoil(rover0,rover0store,waypoint8) 5

samplesoil(rover0,rover0store,waypoint12) 5

samplesoil(rover0,rover0store,waypoint13) 5

samplesoil(rover0,rover0store,waypoint18) 5

samplesoil(rover0,rover0store,waypoint21) 5

samplerock(rover0,rover0store,waypoint5) 5

samplerock(rover0,rover0store,waypoint8) 5

samplerock(rover0,rover0store,waypoint12) 5

samplerock(rover0,rover0store,waypoint13) 5

samplerock(rover0,rover0store,waypoint18) 5

samplerock(rover0,rover0store,waypoint21) 5

calibrate(rover0,camera1,objective1,waypoint21) 1

calibrate(rover0,camera2,objective2,waypoint5) 1

calibrate(rover0,camera3,objective3,waypoint5) 1

calibrate(rover0,camera4,objective4,waypoint12) 1

calibrate(rover0,camera5,objective5,waypoint8) 1

calibrate(rover0,camera8,objective8,waypoint13) 1

takeimage(rover0,waypoint0,objective3,camera3,colour) 4

takeimage(rover0,waypoint0,objective3,camera3,highres) 4

takeimage(rover0,waypoint0,objective3,camera3,lowres) 4

navigate(rover0,waypoint6,waypoint1) 10

navigate(rover0,waypoint6,waypoint2) 15

navigate(rover0,waypoint4,waypoint3) 9

navigate(rover0,waypoint4,waypoint21) 26

navigate(rover0,waypoint6,waypoint5) 9

navigate(rover0,waypoint6,waypoint12) 23

navigate(rover0,waypoint7,waypoint8) 11

navigate(rover0,waypoint7,waypoint11) 8

navigate(rover0,waypoint17,waypoint13) 8

navigate(rover0,waypoint14,waypoint15) 8

navigate(rover0,waypoint17,waypoint18) 16

navigate(rover0,waypoint19,waypoint18) 10

navigate(rover0,waypoint19,waypoint20) 12

navigate(rover0,waypoint19,waypoint21) 13

samplesoil(rover0,rover0store,waypoint4) 5

samplesoil(rover0,rover0store,waypoint6) 5

samplesoil(rover0,rover0store,waypoint7) 5

samplesoil(rover0,rover0store,waypoint14) 5

samplesoil(rover0,rover0store,waypoint17) 5

samplesoil(rover0,rover0store,waypoint19) 5

samplerock(rover0,rover0store,waypoint4) 5

samplerock(rover0,rover0store,waypoint6) 5

samplerock(rover0,rover0store,waypoint14) 5

samplerock(rover0,rover0store,waypoint17) 5

samplerock(rover0,rover0store,waypoint19) 5

drop(rover0,rover0store) 1

calibrate(rover0,camera0,objective0,waypoint19) 1

calibrate(rover0,camera2,objective2,waypoint4) 1

calibrate(rover0,camera3,objective3,waypoint6) 1

calibrate(rover0,camera3,objective3,waypoint4) 1

calibrate(rover0,camera6,objective6,waypoint7) 1

calibrate(rover0,camera7,objective7,waypoint14) 1

calibrate(rover0,camera8,objective8,waypoint17) 1

takeimage(rover0,waypoint21,objective1,camera1,colour) 4

takeimage(rover0,waypoint21,objective1,camera1,highres) 4

takeimage(rover0,waypoint21,objective1,camera1,lowres) 4

takeimage(rover0,waypoint5,objective2,camera2,colour) 4

takeimage(rover0,waypoint5,objective2,camera2,highres) 4

takeimage(rover0,waypoint5,objective2,camera2,lowres) 4

takeimage(rover0,waypoint5,objective3,camera3,colour) 4

takeimage(rover0,waypoint5,objective3,camera3,highres) 4

takeimage(rover0,waypoint5,objective3,camera3,lowres) 4

takeimage(rover0,waypoint12,objective4,camera4,colour) 4

takeimage(rover0,waypoint12,objective4,camera4,highres) 4

takeimage(rover0,waypoint12,objective4,camera4,lowres) 4

takeimage(rover0,waypoint8,objective5,camera5,colour) 4

takeimage(rover0,waypoint8,objective5,camera5,highres) 4

takeimage(rover0,waypoint8,objective5,camera5,lowres) 4

takeimage(rover0,waypoint13,objective8,camera8,colour) 4

takeimage(rover0,waypoint13,objective8,camera8,highres) 4

takeimage(rover0,waypoint13,objective8,camera8,lowres) 4

communicatesoildata(rover0,general,waypoint5,waypoint5,waypoint0) 3

communicatesoildata(rover0,general,waypoint8,waypoint8,waypoint0) 3

communicatesoildata(rover0,general,waypoint12,waypoint12,waypoint0) 3

communicatesoildata(rover0,general,waypoint13,waypoint13,waypoint0) 3

communicatesoildata(rover0,general,waypoint18,waypoint18,waypoint0) 3

communicatesoildata(rover0,general,waypoint21,waypoint21,waypoint0) 3

communicaterockdata(rover0,general,waypoint5,waypoint5,waypoint0) 3

communicaterockdata(rover0,general,waypoint8,waypoint8,waypoint0) 3

communicaterockdata(rover0,general,waypoint12,waypoint12,waypoint0) 3

communicaterockdata(rover0,general,waypoint13,waypoint13,waypoint0) 3

communicaterockdata(rover0,general,waypoint18,waypoint18,waypoint0) 3

communicaterockdata(rover0,general,waypoint21,waypoint21,waypoint0) 3

navigate(rover0,waypoint1,waypoint2) 10

navigate(rover0,waypoint2,waypoint1) 10

navigate(rover0,waypoint1,waypoint6) 10

navigate(rover0,waypoint2,waypoint1) 10

navigate(rover0,waypoint1,waypoint2) 10

navigate(rover0,waypoint2,waypoint6) 15

navigate(rover0,waypoint3,waypoint2) 8

navigate(rover0,waypoint2,waypoint3) 8

navigate(rover0,waypoint11,waypoint7) 8

navigate(rover0,waypoint11,waypoint10) 7

navigate(rover0,waypoint15,waypoint14) 8

navigate(rover0,waypoint15,waypoint16) 8

navigate(rover0,waypoint15,waypoint19) 34

navigate(rover0,waypoint20,waypoint21) 11

samplesoil(rover0,rover0store,waypoint1) 5

samplesoil(rover0,rover0store,waypoint2) 5

samplesoil(rover0,rover0store,waypoint11) 5

samplesoil(rover0,rover0store,waypoint15) 5

samplerock(rover0,rover0store,waypoint2) 5

samplerock(rover0,rover0store,waypoint3) 5

samplerock(rover0,rover0store,waypoint11) 5

samplerock(rover0,rover0store,waypoint15) 5

samplerock(rover0,rover0store,waypoint20) 5

calibrate(rover0,camera2,objective2,waypoint3) 1

calibrate(rover0,camera6,objective6,waypoint11) 1

calibrate(rover0,camera7,objective7,waypoint15) 1

takeimage(rover0,waypoint19,objective0,camera0,colour) 4

takeimage(rover0,waypoint19,objective0,camera0,highres) 4

takeimage(rover0,waypoint19,objective0,camera0,lowres) 4

takeimage(rover0,waypoint4,objective2,camera2,colour) 4

takeimage(rover0,waypoint4,objective2,camera2,highres) 4

takeimage(rover0,waypoint4,objective2,camera2,lowres) 4

takeimage(rover0,waypoint6,objective3,camera3,colour) 4

takeimage(rover0,waypoint6,objective3,camera3,highres) 4

takeimage(rover0,waypoint6,objective3,camera3,lowres) 4

takeimage(rover0,waypoint4,objective3,camera3,colour) 4

takeimage(rover0,waypoint4,objective3,camera3,highres) 4

takeimage(rover0,waypoint4,objective3,camera3,lowres) 4

takeimage(rover0,waypoint7,objective6,camera6,colour) 4

takeimage(rover0,waypoint7,objective6,camera6,highres) 4

takeimage(rover0,waypoint7,objective6,camera6,lowres) 4

takeimage(rover0,waypoint14,objective7,camera7,colour) 4

takeimage(rover0,waypoint14,objective7,camera7,highres) 4

takeimage(rover0,waypoint14,objective7,camera7,lowres) 4

takeimage(rover0,waypoint17,objective8,camera8,colour) 4

takeimage(rover0,waypoint17,objective8,camera8,highres) 4

takeimage(rover0,waypoint17,objective8,camera8,lowres) 4

communicatesoildata(rover0,general,waypoint8,waypoint12,waypoint0) 3

communicatesoildata(rover0,general,waypoint13,waypoint12,waypoint0) 3

communicatesoildata(rover0,general,waypoint18,waypoint21,waypoint0) 3

communicaterockdata(rover0,general,waypoint8,waypoint12,waypoint0) 3

communicaterockdata(rover0,general,waypoint13,waypoint12,waypoint0) 3

communicaterockdata(rover0,general,waypoint18,waypoint21,waypoint0) 3

communicateimagedata(rover0,general,objective2,colour,waypoint5,waypoint0) 3

communicateimagedata(rover0,general,objective3,colour,waypoint5,waypoint0) 3

communicateimagedata(rover0,general,objective2,highres,waypoint5,waypoint0) 3

communicateimagedata(rover0,general,objective3,highres,waypoint5,waypoint0) 3

communicateimagedata(rover0,general,objective2,lowres,waypoint5,waypoint0) 3

communicateimagedata(rover0,general,objective3,lowres,waypoint5,waypoint0) 3

communicateimagedata(rover0,general,objective3,colour,waypoint8,waypoint0) 3

communicateimagedata(rover0,general,objective5,colour,waypoint8,waypoint0) 3

communicateimagedata(rover0,general,objective3,highres,waypoint8,waypoint0) 3

communicateimagedata(rover0,general,objective5,highres,waypoint8,waypoint0) 3

communicateimagedata(rover0,general,objective3,lowres,waypoint8,waypoint0) 3

communicateimagedata(rover0,general,objective5,lowres,waypoint8,waypoint0) 3

communicateimagedata(rover0,general,objective3,colour,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective4,colour,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective3,highres,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective4,highres,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective3,lowres,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective4,lowres,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective3,colour,waypoint13,waypoint0) 3

communicateimagedata(rover0,general,objective8,colour,waypoint13,waypoint0) 3

communicateimagedata(rover0,general,objective3,highres,waypoint13,waypoint0) 3

communicateimagedata(rover0,general,objective8,highres,waypoint13,waypoint0) 3

communicateimagedata(rover0,general,objective3,lowres,waypoint13,waypoint0) 3

communicateimagedata(rover0,general,objective8,lowres,waypoint13,waypoint0) 3

communicateimagedata(rover0,general,objective3,colour,waypoint18,waypoint0) 3

communicateimagedata(rover0,general,objective3,highres,waypoint18,waypoint0) 3

communicateimagedata(rover0,general,objective3,lowres,waypoint18,waypoint0) 3

communicateimagedata(rover0,general,objective1,colour,waypoint21,waypoint0) 3

communicateimagedata(rover0,general,objective3,colour,waypoint21,waypoint0) 3

communicateimagedata(rover0,general,objective1,highres,waypoint21,waypoint0) 3

communicateimagedata(rover0,general,objective3,highres,waypoint21,waypoint0) 3

communicateimagedata(rover0,general,objective1,lowres,waypoint21,waypoint0) 3

communicateimagedata(rover0,general,objective3,lowres,waypoint21,waypoint0) 3

navigate(rover0,waypoint10,waypoint9) 7

navigate(rover0,waypoint10,waypoint11) 7

navigate(rover0,waypoint16,waypoint15) 8

navigate(rover0,waypoint16,waypoint17) 8

samplesoil(rover0,rover0store,waypoint10) 5

samplesoil(rover0,rover0store,waypoint16) 5

samplerock(rover0,rover0store,waypoint10) 5

calibrate(rover0,camera6,objective6,waypoint10) 1

takeimage(rover0,waypoint3,objective2,camera2,colour) 4

takeimage(rover0,waypoint3,objective2,camera2,highres) 4

takeimage(rover0,waypoint3,objective2,camera2,lowres) 4

takeimage(rover0,waypoint11,objective6,camera6,colour) 4

takeimage(rover0,waypoint11,objective6,camera6,highres) 4

takeimage(rover0,waypoint11,objective6,camera6,lowres) 4

takeimage(rover0,waypoint15,objective7,camera7,colour) 4

takeimage(rover0,waypoint15,objective7,camera7,highres) 4

takeimage(rover0,waypoint15,objective7,camera7,lowres) 4

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communicatesoildata(rover0,general,waypoint17,waypoint18,waypoint0) 3

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communicaterockdata(rover0,general,waypoint17,waypoint18,waypoint0) 3

communicaterockdata(rover0,general,waypoint19,waypoint18,waypoint0) 3

communicaterockdata(rover0,general,waypoint4,waypoint21,waypoint0) 3

communicaterockdata(rover0,general,waypoint5,waypoint21,waypoint0) 3

communicaterockdata(rover0,general,waypoint19,waypoint21,waypoint0) 3

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communicateimagedata(rover0,general,objective8,colour,waypoint12,waypoint0 3

communicateimagedata(rover0,general,objective5,highres,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective8,highres,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective5,lowres,waypoint12,waypoint0) 3

communicateimagedata(rover0,general,objective8,lowres,waypoint12,waypoint0) 3

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navigate(rover0,waypoint9,waypoint13) 14

samplerock(rover0,rover0store,waypoint9) 5

calibrate(rover0,camera5,objective5,waypoint9) 1

takeimage(rover0,waypoint10,objective6,camera6,colour) 4

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takeimage(rover0,waypoint10,objective6,camera6,lowres) 4

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communicateimagedata(rover0,general,objective6,lowres,waypoint8,waypoint0) 3

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communicateimagedata(rover0,general,objective2,highres,waypoint21,waypoint0) 3

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communicateimagedata(rover0,general,objective2,lowres,waypoint21,waypoint0) 3

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takeimage(rover0,waypoint9,objective5,camera5,highres) 4

takeimage(rover0,waypoint9,objective5,camera5,lowres) 4

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communicaterockdata(rover0,general,waypoint15,waypoint21,waypoint0) 3

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communicateimagedata(rover0,general,objective6,lowres,waypoint12,waypoint0) 3

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communicateimagedata(rover0,general,objective8,lowres,waypoint21,waypoint0) 3

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communicateimagedata(rover0,general,objective7,lowres,waypoint13,waypoint0) 3

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communicatesoildata(rover0,general,waypoint8,waypoint18,waypoint0) 3

communicatesoildata(rover0,general,waypoint10,waypoint18,waypoint0) 3

communicatesoildata(rover0,general,waypoint11,waypoint18,waypoint0) 3

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communicateimagedata(rover0,general,objective6,highres,waypoint18,waypoint0) 3

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communicatedsoildata(waypoint8) 25

communicatedsoildata(waypoint12) 25

communicatedsoildata(waypoint13) 25

communicatedsoildata(waypoint18) 25

communicatedsoildata(waypoint21) 25

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communicatedrockdata(waypoint13) 30

communicatedrockdata(waypoint18) 30

communicatedrockdata(waypoint21) 30

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communicatedimagedata(objective2,highres) 25

communicatedimagedata(objective3,highres) 25

communicatedimagedata(objective2,lowres) 20

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communicatedimagedata(objective5,colour) 22

communicatedimagedata(objective5,lowres) 20

communicatedimagedata(objective4,colour) 22

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communicatedimagedata(objective4,lowres) 20

communicatedimagedata(objective8,lowres) 20

communicatedimagedata(objective1,colour) 22

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communicatedimagedata(objective1,lowres) 20

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communicatedsoildata(waypoint7) 25

communicatedsoildata(waypoint17) 25

communicatedsoildata(waypoint19) 25

communicatedsoildata(waypoint4) 25

communicatedrockdata(waypoint6) 30

communicatedrockdata(waypoint17) 30

communicatedrockdata(waypoint19) 30

communicatedrockdata(waypoint4) 30

communicatedrockdata(waypoint20) 30

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communicatedimagedata(objective0,highres) 25

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communicatedsoildata(waypoint2) 25

communicatedsoildata(waypoint11) 25

communicatedsoildata(waypoint14) 25

communicatedsoildata(waypoint15) 25

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communicatedrockdata(waypoint11) 30

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communicatedrockdata(waypoint15) 30

communicatedsoildata(waypoint10) 25

communicatedsoildata(waypoint16) 25
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communicatedrockdata(waypoint9) 30

communicatedrockdata(waypoint10) 30

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communicatedimagedata(objective7,lowres) 20
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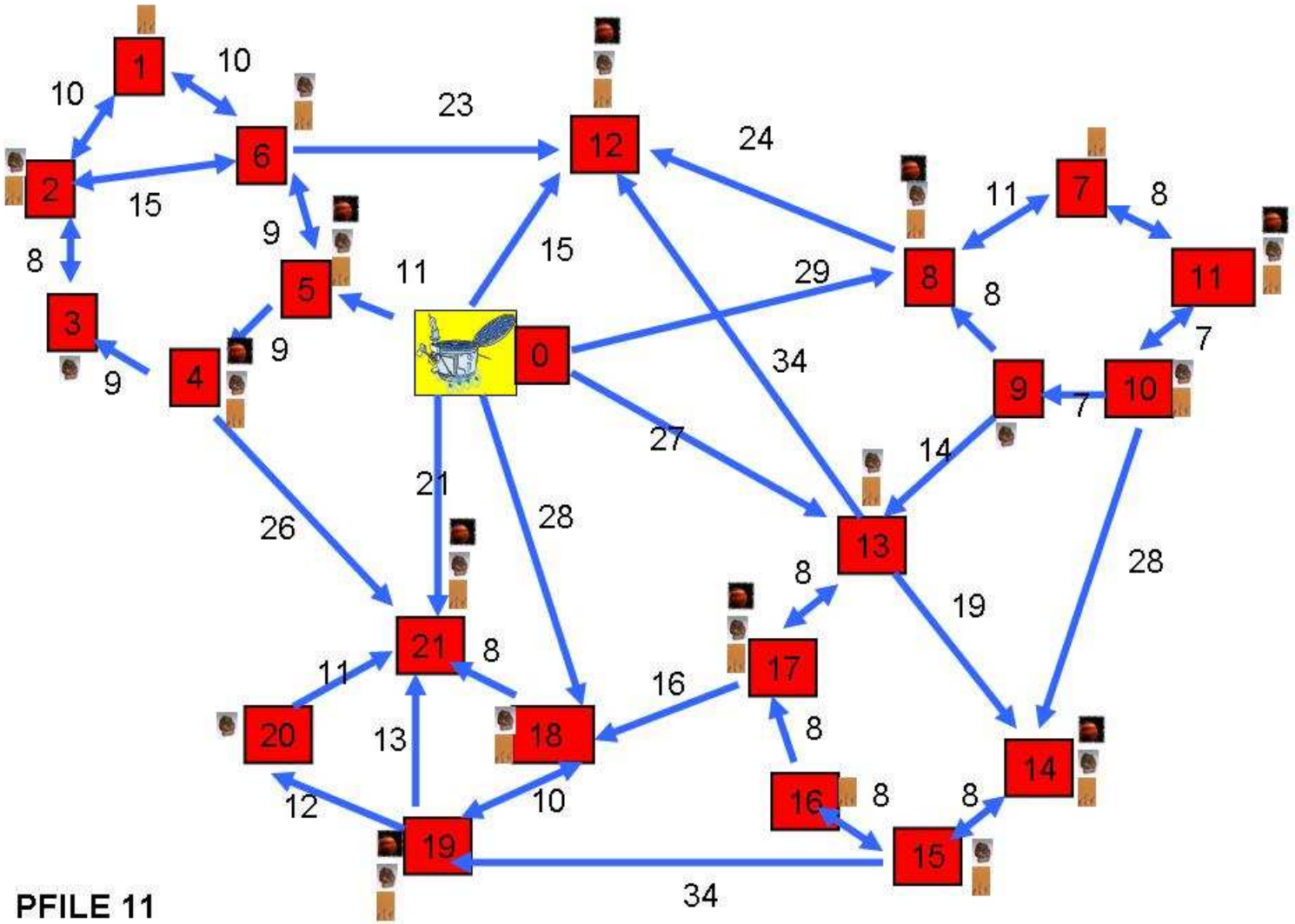

Figure 32. Graphical view of rover problem 11.